MATHEMATICS-X

MODULE-3

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QUADRATIC EQUATIONS

INTRODUCTION

When a polynomial f(x) is equated to zero, we get an equation which is known as a polynomial equation. If f(x) is a linear polynomial then f(x) = 0 is called a linear equation. For example, $3x - 2 = 0$, $4t + \frac{3}{5} = 0$ etc. are linear equations. If f(x) is a quadratic polynomial i.e., $f(x) = ax^2 + bx + c$, $a \neq 0$, then $f(x) = 0$ i.e., $ax^2 + bx + c = 0$, $a \neq 0$ is called a quadratic equation. Such equations arise in many real life situations. In this chapter, we will learn about quadratic equations and various ways of finding their zeros or roots. In the end of the chapter, we will also discuss some applications of quadratic equations in daily life situations.

HISTORICAL FACTS

On clay tables dated between 1800 BC and 1600 BC, the ancient Babylonians left the earliest evidence of the discovery of quadratic equations, and also gave early methods for solving them.

Indian mathematician Baudhayana who wrote a Sulba Sutra in ancient India circa 8th century BC first used quadratic equations of the form : $ax^2 = c$ and $ax^2 + bx = c$ and also gave methods for solving them.

Babylonian mathematicians from circa 400 BC and Chinese mathematicians from circa 200 BC used the method of completing the square to solve quadratic equations with positive roots, but did not have a general formula.

Euclid, a Greek mathematician, produced a more abstract geometrical method around 300 BC.

The first mathematician to have found negative solutions with the general algebraic formula was Brahmagupta (India, 7th century). He gave the first explicit (although still not completely general) solution of the quadratic equation $ax^2 + bx = c$ as follows :

"To the absolute number multiplied by four times the [coefficient of the] square, add the square of the [coefficient of the] middle term; the square root of the same, less the [coefficient of the] middle term, being divided by twice the [coefficient of the] square is the value."

This is equivalent to : $x = \frac{\sqrt{4ac + b^2} - b}{2a}$

Muhammad ibn Musa al-Kwarizmi (Persia, 9th century) developed a set of formulae that worked for positive solutions.

Bhaskara II (1114-1185), an Indian mathematician-astronomer, solved quadratic equations with more than one unknown and is considered the originator of the equation.

Shridhara (India, 9th century) was one of the first mathematicians to give a general rule for solving a quadratic equation.

BASIC CONCEPT

1. Every algebraic polynomial of second degree is called a quadratic polynomial.

   For Example :

   (i) $3x^2+5x+7$ (ii) $8x^2-6x$

   (iii) $5x^2 - 7$ (iv) $\sqrt{2}\,x^2+6x-\sqrt{3}$ and so on.

2. The general form of quadratic polynomial is $ax^2 + bx + c$; where a, b, c are real numbers, $a \neq 0$ and x is variable.

   For a particular quadratic polynomial the values of a, b, c are constant and for this reason a, b and c are also called real constants. For example, in quadratic polynomial $3x^2 - 5x + 8$; 3, –5 and 8 are constant where as x is variable.

Value of a Quadratic Polynomial

The value of a quadratic polynomial $ax^2 + bx + c$

(i) at $x = \alpha$ is $= a(\alpha)^2 + b(\alpha) + c = a\alpha^2 + b\alpha + c$

(ii) at $x = \beta$ is $a\beta^2 + b\beta + c$

(iii) at $x = 5$ is $= a\,(5)^2 + b(5) + c = 25a^2 + 5a + c$ and so on.

In the same way :

(i) Value of $5x^2 - 3x + 4$ at $x = 2$ is $= 5(2)^2 - 3(2) + 4 = 20 - 6 + 4 = 18$

(ii) Value of $x^2 - 8x - 15$ at $x = -1$ is $= (-1)^2 - 8(-1) - 15 = 1 + 8 - 15 = -6$

(iii) Value of $7x^2 - 4$ at $x = \frac{2}{3}$ is $= 7\left(\frac{2}{3}\right)^2 - 4$

$= 7 \times \frac{4}{9} - 4 = \frac{28-36}{9} = \frac{-8}{9}$ and so on.

Zeros of a Quadratic Polynomial

The value of the polynomial $x^2 - 7x + 10$ at :

(i) $x = 1$ is $(1)^2 - 7 \times 1 + 10 = 1 - 7 + 10 = 4$

(ii) $x = 2$ is $(2)^2 - 7 \times 2 + 10 = 4 - 14 + 10 = 0$

(iii) $x = 3$ is $(3)^2 - 7 \times 3 + 10 = 9 - 21 + 10 = -2$

(iv) $x = 5$ is $(5)^2 - 7 \times 5 + 10 = 25 - 35 + 10 = 0$

and so on.

It is observed here that for $x = 2$ and $x = 5$; the value of polynomial $x^2 - 7x + 10$ is zero. These two values of x are called zeros of the polynomial.

Thus, if for $x = \alpha$, where α is a real number, the value of given quadratic polynomial is zero; the real number α is called zero of the quadratic polynomial.

SUMMARY OF THE CHAPTER

BASIC CONCEPTS AND IMPORTANT RESUTLS

* **Quadratic polynomial**

An algebraic expression of the form $ax^2 + bx + c$, where a, b, c are real numbers and $a \neq 0$, is called a quadratic polynomial in the variable x.

Zeroes of a quadratic polynomial.

Let $p(x) = ax^2 + bx + c$, $a \neq 0$, be a quadratic polynomial, then a real number 'α' is called a zero of the quadratic polynomial if and only if $p(\alpha) = 0$ i.e. if and only if $a\alpha^2 + b\alpha + c = 0$.

* **Quadratic equation**

An equation of the form $ax^2 + bx + c =$, where a, b, c are real numbers and $a \neq 0$, is called a quadratic equation in the variable x.

Note. Simplify the given equation before deciding whether it is quadratic or not.

* **Roots (or solutions) of a quadratic equation**

Let $ax^2 + bx + c = 0$, $a \neq 0$, be a quadratic equation, then a real number α is called a root (or solution) of the quadratic equation if and only if it satisfies the equation i.e. if and only if $a\alpha^2 + b\alpha + c = 0$.

Hence, the zeroes of the quadratic polynomial $ax^2 + bx + c$ and the roots of the quadratic equation $ax^2 + bx + c = 0$ are same.

Thus, α is a zero of the quadratic polynomial $ax^2 + bx + c$ if and only if α is a root of the quadratic equation $ax^2 + bx + c = 0$.

Also, by factor theorem, $x - \alpha$ is a factor of $ax^2 + bx + c$. If and only if $a\alpha^2 + b\alpha + c = 0$. It follows that :

(i) $x - \alpha$ is a factor of $ax^2 + bx + c$ if and only if α is a zero of the quadratic polynomial $ax^2 + bx + c$.

(ii) $x - \alpha$ is a factor of $ax^2 + bx + c$ if and only if α is a root of the quadratic equation $ax^2 + bx + c = 0$.

* **Number of roots of a quadratic equation**

A quadratic equation $ax^2 + bx + c = 0$, where a, , c are real numbers, $a \neq 0$, has

(i) either two distinct real roots

(ii) or two equal real roots i.e. one real root (repeated twice)

(iii) or no real roots. Thus, a quadratic equation cannot have more than two roots.

* **Zero-product rule**

If a and b are two real numbers or algebraic expressions and if ab = 0, then either a = 0 or b = 0 or both a = 0 and b = 0.

* **Solving a quadratic equation by factorisation**

Procedure :

1. Resolve the quadratic polynomial $ax^2 + bx + c$ into the product of two linear factors.

2. Put each linear factor equal to zero.

3. Solve the resulting linear equations.

* **Solving a quadratic equation by completing the square**

Procedure :

Let $ax^2 + bx + c = 0$, where a, b, c are real numbers and $a \neq 0$, be the given quadratic equation. Dividing by a, we get

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0 \quad \Rightarrow \quad x^2 + \frac{b}{a}x = -\frac{c}{a}.$$

Adding ($\frac{1}{2}$ coeff of x)^2 i.e. $\left(\frac{b}{2a}\right)^2$ to both sides, we get $x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2 - \frac{c}{a}$

$$\Rightarrow \quad \left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}.$$

If $b^2 - 4ac \geq 0$, the by taking the square roots, we get $x + \frac{b}{2a} = \pm \frac{\sqrt{b^2 - 4ac}}{2a}$

$$\Rightarrow \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Thus, a quadratic equation can be solved by the method of completing the square.

* **Solving a quadratic equation by using quadratic formula Procedure :**

Let $ax^2 + bx + c = 0$, where a, b, c are real numbers and $a \neq 0$, be the given quadratic equation.

Find (discriminant) $D = b^2 - 4ac$. Three cases arise :

Case I. If D > 0, then the given quadratic equation has two real and different roots. These roots are given by

$$x = \frac{-b \pm \sqrt{D}}{2a} \text{ i.e. } x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}, \frac{-b - \sqrt{b^2 - 4ac}}{2a}.$$

Case II. If D = 0, then the given quadratic equation has two equal real roots i.e. it has one real root which is repeated twice. This repeated root is given by $x = -\frac{b}{2a}$.

Case III. If D < 0, then the given equation has no real roots. Since we are concerned only with real roots, so in this case we may say that the given equation has no roots.

* **Nature of roots of a quadratic equation**

Let $ax^2 + bx + c = 0$, where a, b, c are real numbers, $a \neq 0$, be the given quadratic equation. Then the given equation has

(i) Two real and different roots if and only if $b^2 > 4ac$

(ii) Two real and equal roots if and only if $b^2 = 4ac$

(iii) Two real roots if and only if $b^2 \geq 4ac$

(iv) no real roots if and only if $b^2 < 4ac$.

In particular, if a, b, c are rational numbers and $b^2 - 4ac > 0$, then the given quadratic equation has

(i) Two rational and different roots if and only if $\sqrt{b^2 - 4ac}$ is a rational number

(ii) Two irrational and different roots if and only if $\sqrt{b^2 - 4ac}$ is an irrational number. Moreover, irrational roots occur in conjugate pairs i.e. if $p + \sqrt{q}$, where p, q are rational numbers, $q > 0$ and $\sqrt{q}$ is irrational, is one root of the given quadratic equation, then the other root is $p - \sqrt{q}$.

* A quadratic polynomial $ax^2 + bx + c$ is expressible as the product of two linear factors only when $b^2 \geq 4ac$.

* **Relations between the roots and the coefficients of a quadratic equation :**

If α, β are roots of the quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$, then

(i) $\alpha + \beta = -\frac{b}{a} = -\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$

(ii) $\alpha\beta = \frac{c}{a} = \frac{\text{constant term}}{\text{coefficient of } x^2}$.

* **To obtain a quadratic equation whose roots are given**

A quadratic equation whose roots are α and β is

$(x - \alpha)(x - \beta) = 0$ i.e. $x^2 - (\alpha + \beta)x + \alpha\beta = 0$

or $x^2 - (\text{sum of roots})x + \text{product of roots} = 0$.

* **Applications of quadratic equations**

Quadratic equations are used to solve a number of applied problems that arise in our daily life. Due to the wide variety of applied problems, there is no single technique that works in all cases. However, the following general suggestions should prove helpful :

(i) Read the statement of the problem carefully and determine what quantity (or quantities) must be found.

(ii) Represent the unknown quantity (or quantities) by a variable (or variables).

(iii) Identify the relationships existing in the problem and determine which expressions are equal and write an equation (or equations).

(iv) Form the quadratic equation and solve the resulting equation.

(v) Interpret the solution of the equation.

Note. Check the answers obtained, by determining whether they fulfill the conditions of the original problem. It may happen that out of the two roots of the quadratic equation, only one satisfies the conditions of the problem, reject the other root.

SOLVED PROBLEMS

Ex.1 Check whether the following are quadratic equations : **[NCERT]**

(i) $(x + 1)^2 = 2(x - 3)$ (ii) $(x - 2)(x + 1) = (x - 1)(x + 3)$

(iii) $(x - 3)(2x + 1) = x(x + 5)$

Sol. (i) Here, the given equation is $(x + 1)^2 = 2(x - 3)$

$\Rightarrow x^2 + 2x + 1 = 2x - 6$

$\Rightarrow x^2 + 2x - 2x + 1 + 6 = 0$

$\Rightarrow x^2 + 7 = 0 \Rightarrow x^2 + 0.x + 7 = 0$, which is of the form $ax^2 + bx + c = 0$

Hence, $(x + 1)^2 = 2(x - 3)$ is a quadratic equation.

(ii) Here, the given equation is

$(x - 2)(x + 1) = (x - 1)(x + 3)$

$\Rightarrow x^2 + x - 2x - 2 = x^2 + 3x - x - 3$

$\Rightarrow x^2 - x^2 - x - 2x - 2 + 3 = 0 \Rightarrow -3x + 1 = 0$,

Which is not of the form $ax^2 + bx + c = 0$

Hence, $(x - 2)(x + 1) = (x - 1)(x + 3)$ is not a quadratic equation

(iii) Here, the given equation is

$(x - 3)(2x + 1) = x(x + 5)$

$\Rightarrow 2x^2 + x - 6x - 3 = x^2 + 5x$

$\Rightarrow 2x^2 - x^2 - 5x - 5x - 3 = 0$

$\Rightarrow x^2 - 10x - 3 = 0$,

Which is of the form $ax^2 + bx + c = 0$.

Hence, $(x - 3)(2x + 1) = x(x + 5)$ is a quadratic equation.

Ex.2 In each of the following, determine whether the given values are the solution of the given equation or not:

(i) $\frac{2}{x^2} - \frac{5}{x} + 2 = 0$; $x = 5$, $x = \frac{1}{2}$

(ii) $a^2x^2 - 3abx + 2b^2 = 0$; $x = \frac{a}{b}, x = \frac{b}{a}$

Sol. (i) Putting $x = 5$ and $x = \frac{1}{2}$ in the given equation.

$\frac{2}{(5)^2} - \frac{5}{5} + 2$ and $\frac{2}{\left(\frac{1}{2}\right)^2} - \frac{5}{\left(\frac{1}{2}\right)} + 2$

$\Rightarrow \frac{2}{25} - 1 + 2$ and $\frac{2}{\frac{1}{4}} - \frac{5}{\frac{1}{2}} + 2$

$\Rightarrow \frac{2}{25} + 1$ and $8 - 10 + 2 \Rightarrow \frac{27}{25}$ and 0

i.e., $x = 5$ does not satisfy but $x = \frac{1}{2}$ satisfies the given equation.

Hence, $x = 5$ is not a solution but $x = \frac{1}{2}$ is a solution of $\frac{2}{x^2} - \frac{5}{x} + 2 = 0$.

(ii) Putting $x = \frac{a}{b}$ and $x = \frac{b}{a}$ in the given equation.

$a^2\left(\frac{a}{b}\right)^2 - 3ab\left(\frac{a}{b}\right) + 2b^2$ and $a^2\left(\frac{b}{a}\right)^2 - 3ab\left(\frac{b}{a}\right) + 2b^2$

$\Rightarrow \frac{a^4}{b^2} + 2b^2 - 3a^2$ and 0

i.e., $x = \frac{a}{b}$ does not satisfy but $x = \frac{b}{a}$ satisfies the given equation.

Hence $x = \frac{b}{a}$ is a solution but $x = \frac{a}{b}$ is not a solution of $a^2x^2 - 3abx + 2b^2 = 0$.

Ex.3 Find the values of p and q for which $x = \frac{3}{4}$ and $x = -2$ are the roots of the equation $px^2 + qx - 6 = 0$.

Sol. Since $x = \frac{3}{4}$ and $x = -2$ are the roots of the equation $px^2 + qx - 6 = 0$.

$\therefore p\left(\frac{3}{4}\right)^2 + q\left(\frac{3}{4}\right) - 6 = 0$ and $p(-2)^2 + q(-2) - 6 = 0$

$\Rightarrow p\times\frac{9}{16} + q\times\frac{3}{4} - 6 = 0$ and $4p - 2q - 6 = 0$

$\Rightarrow \frac{9p + 12q - 96}{16} = 0$ and $4p - 2q - 6 = 0$

$\Rightarrow 9p + 12q - 96 = 0$ and $4p - 2q - 6 = 0$

$\Rightarrow 3p + 4q - 32 = 0$ ...(i)

and $2p - q - 3 = 0$ ...(ii)

Multiplying (ii) by 4,

we get $8p - 4q - 12 = 0$ ...(iii)

Adding (i) and (iii), we get $p = 4$

Putting the value of p in equation (ii), we get

$2\times4 - q - 3 = 0 \Rightarrow q = 5$ Hence, $p = 4$, $q = 5$.

Ex.4 Solve the following quadratic equation by factorization method $x^2 - 2ax + a^2 - b^2 = 0$

Sol. Factors of the constant term $a^2 - b^2$ are $(a - b)$ & $(a + b)$ also coefficient of the middle term $= -2a = -[(a - b) + (a + b)]$

$\Rightarrow x^2 - 2ax + a^2 - b^2 = 0$

$\Rightarrow x^2 - \{(a - b) + (a + b)\}x + (a + b)(a - b) = 0$

$\Rightarrow x^2 - (a - b)x - (a + b)x + (a - b)(a + b) = 0$

$\Rightarrow x[x - (a - b)] - (a + b)[x - (a - b)] = 0$

$\Rightarrow [x - (a - b)][x - (a + b)] = 0$

$x - (a - b) = 0$ or $x - (a + b) = 0$

$x = a - b$, $x = a + b$

Ex.5 Solve the quadratic equation $5x^2 = -16x - 12$ by factorisation method.

Sol. $5x^2 = -16x - 12$

$5x^2 + 16x + 12 = 0$

$5x^2 + 10x + 6x + 12 = 0$

$5x(x + 2) + 6(x + 2) = 0$

$(x + 2)(5x + 6) = 0$

$x + 2 = 0 \quad \Rightarrow x = -2$

or $5x + 6 = 0 \quad \Rightarrow x = \frac{-6}{5}$

Ex.6 Solve : $9x^2 - 15x + 6 = 0$

Sol. Here, $9x^2 - 15x + 6 = 0$

$\Rightarrow x^2 - \frac{15}{9}x + \frac{6}{9} = 0$

[Dividing throughout by 9]

$\Rightarrow x^2 - \frac{5}{3}x + \frac{2}{3} = 0 \quad \Rightarrow \quad x^2 - \frac{5}{3}x = -\frac{2}{3}$

[$\because$ Shifting the constant term on RHS)

$$\Rightarrow \quad x^2 - 2\left(\frac{5}{6}\right)x + \left(\frac{5}{6}\right)^2 = \left(\frac{5}{6}\right)^2 - \frac{2}{3}$$

[$\because$ Adding square of half of coefficient of x on both sides]

$$\Rightarrow \quad \left(x-\frac{5}{6}\right)^2 = \frac{25}{36} - \frac{2}{3} \Rightarrow \left(x-\frac{5}{6}\right)^2 = \frac{25-24}{36} \Rightarrow \left(x-\frac{5}{6}\right)^2 = \frac{1}{36} \Rightarrow \quad x-\frac{5}{6} = \pm\frac{1}{6}$$

[Taking square root of both sides]

$$\Rightarrow \quad x = \frac{5}{6} \pm \frac{1}{6} \qquad \Rightarrow x = \frac{5}{6} + \frac{1}{6} = 1$$

$$\text{or,} \quad x = \frac{5}{6} - \frac{1}{6} = \frac{4}{6} = \frac{2}{3} \Rightarrow \quad x = 1 \text{ or, } x = \frac{2}{3}$$

Ex.7 Solve the equation $x^2 - (\sqrt{3} + 1)x + \sqrt{3} = 0$ by the method of completing the square.

Sol. We have,

$$x^2 - (\sqrt{3} + 1)x + \sqrt{3} = 0$$

$$\Rightarrow \quad x^2 - (\sqrt{3} + 1)x = -\sqrt{3}$$

$$\Rightarrow \quad x^2 - 2\left(\frac{\sqrt{3}+1}{2}\right)x + \left(\frac{\sqrt{3}+1}{2}\right)^2$$

$$= \quad -\sqrt{3} + \left(\frac{\sqrt{3}+1}{2}\right)^2$$

$$\Rightarrow \quad \left(x-\frac{\sqrt{3}+1}{2}\right)^2 = \frac{-4\sqrt{3}+(\sqrt{3}+1)^2}{4}$$

$$\Rightarrow \quad \left(x-\frac{\sqrt{3}+1}{2}\right)^2 = \left(\frac{\sqrt{3}-1}{2}\right)^2$$

$$\Rightarrow \quad x-\frac{\sqrt{3}+1}{2} = \pm\frac{\sqrt{3}-1}{2}$$

$$\Rightarrow \quad x = \frac{\sqrt{3}+1}{2} \pm \frac{\sqrt{3}-1}{2} \qquad \Rightarrow \quad x = \sqrt{3}, 1$$

Hence, the roots are $\sqrt{3}$ and 1.

Ex.8 Solve the quadratic equation $x^2 - 6x + 4 = 0$ by using quadratic formula (Sreedharacharya's Rule).

Sol. On comparing the given equation $x^2 - 6x + 4 = 0$ with the standard quadratic equation $ax^2 + bx + c = 0$, we get $a = 1, b = -6, c = 4$

Hence the required roots are

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(4)}}{2(1)} = \frac{6 \pm \sqrt{36-16}}{2}$$

$$= \frac{6 \pm \sqrt{20}}{2} = \frac{6 \pm \sqrt{4\times 5}}{2} = \frac{2(3 \pm \sqrt{5})}{2} = 3 \pm \sqrt{5}$$

Thus the roots of the equation are $3 + \sqrt{5}$ & $3 - \sqrt{5}$.

Ex.9 Find the nature of the roots of the following equations. If the real roots exist, find them. **[NCERT]**

(i) $2x^2 - 6x + 3 = 0$ (ii) $2x^2 - 3x + 5 = 0$

Sol. (i) The given equation $2x^2 - 6x + 3 = 0$

comparing it with $ax^2 + bx + c = 0$, we get

$a = 2$, $b = -6$ and $c = 3$.

$\therefore$ Discriminant, $D = b^2 - 4ac = (-6)^2 - 4.2.3$

$= 36 - 24 = 12 > 0$

$\because$ $D > 0$, roots are real and unequal.

Now, by quadratic formula, $x = \frac{-b \pm \sqrt{D}}{2a} = \frac{6 \pm \sqrt{12}}{2 \times 2} = \frac{6 \pm 2\sqrt{3}}{4} = \frac{3 \pm \sqrt{3}}{2}$

Hence the roots are $x = \frac{3+\sqrt{3}}{2}, \frac{3-\sqrt{3}}{2}$

(ii) Here, the given equation is $2x^2 - 3x + 5 = 0$;

Comparing it with $ax^2 + bx + c = 0$, we get

$a = 2$, $b = -3$ and $c = 5$

$\therefore$ Discriminant, $D = b^2 - 4ac = 9 - 4 \times 2 \times 5 = 9 - 40 = -31$

$\because$ $D < 0$, the equation has no real roots.

Ex.10 Find the value of k for each of the following quadratic equations, so that they have real and equal roots:

(i) $9x^2 + 8kx + 16 = 0$

(ii) $(k + 1)x^2 - 2(k - 1)x + 1 = 0$

Sol. (i) The given equation $9x^2 + 8kx + 16 = 0$

comparing it with $ax^2 + bx + c = 0$, we get

$a = 9$, $b = 8k$ and $c = 16$.

$\therefore$ Discriminant, $D = b^2 - 4ac = (8k)^2 - 4 \times 9 \times 16 = 64k^2 - 576$

Since roots are real and equal, so

$D = 0 \Rightarrow 64k^2 - 576 = 0 \Rightarrow 64k^2 = 576$

$\Rightarrow k^2 = \frac{576}{64} = 9 \quad \Rightarrow k = \pm 3$ Hence, $k = 3, -3$

(ii) The given equation is $(k + 1)x^2 - 2(k - 1)x + 1 = 0$

comparing it with $ax^2 + bx + c = 0$, we get

$a = (k + 1)$, $b = -2(k - 1)$ and $c = 1$

$\therefore$ Discriminant, $D = b^2 - 4ac = 4(k - 1)^2 - 4(k + 1) \times 1$

$= 4(k^2 - 2k + 1) - 4k - 4$

$= 4k^2 - 8k + 4 - 4k - 4 = 4k^2 - 12k$

Since roots are real and equal, so

$D = 0 \Rightarrow 4k^2 - 12k = 0$

$\Rightarrow 4k(k - 3) = 0 \Rightarrow$ either $4k = 0$ or $k - 3 = 0$

$\Rightarrow k = 0$ or $k = 3$ Hence, $k = 0, 3$.

Ex.11 Find the set of values of k for which the equation $kx^2 + 2x + 1$ has distinct real roots.

Sol. The given equation is $kx^2 + 2x + 1 = 0$

$\therefore$ $D = (4 - 4 \times k \times 1) = 4 - 4k$.

For distinct and real roots, we must have, $D > 0$.

Now, $D > 0 \Leftrightarrow (4 - 4k) > 0 \Leftrightarrow 4 > 4k$

$\Leftrightarrow 4k < 4 \Leftrightarrow k < 1$.

$\therefore$ Required set $= \{k \in R : k < 1\}$

Ex.12 Find the values of k for which the equation $5x^2 - kx + 4 = 0$ has real roots.

Sol. The given equation is $5x^2 - kx + 4 = 0$

$\therefore$ $D = k^2 - 4 \times 5 \times 4 = k^2 - 80$

For real roots, we must have, $D \geq 0$

Now, $D \geq 0 \Leftrightarrow k^2 - 80 \geq 0 \Leftrightarrow k^2 \geq 80$

$\Leftrightarrow k \geq \sqrt{80}$ or $k \leq -\sqrt{80} \Leftrightarrow k \geq 4\sqrt{5}$ or $k \leq -4\sqrt{5}$.

Ex.13 Form the quadratic equation in each of the following cases when the roots are :

(i) $2 + \sqrt{5}$ and $2 - \sqrt{5}$ (ii) a and $\frac{1}{a}$

Sol. (i) Here roots are $\alpha = 2 + \sqrt{5}$ and $\beta = 2 - \sqrt{5}$

$\therefore$ Sum of roots $= \alpha + \beta = (2 + \sqrt{5}) + (2 - \sqrt{5})$

$\therefore$ $\alpha + \beta = 4$

and product of the roots $= \alpha \cdot \beta = (2 + \sqrt{5})(2 - \sqrt{5}) = 4 - 5 = -1$

$\therefore$ $\alpha\beta = -1$

$\therefore$ Required equation is

x^2 – (sum of roots) x + product of roots = 0

or $x^2 - (\alpha + \beta)x + \alpha \cdot \beta = 0$

or $x^2 - (4)x + (-1) = 0$

$\therefore$ $x^2 - 4x - 1 = 0$

(ii) Here roots are a and $\frac{1}{a}$

$\therefore$ $\alpha + \beta = a + \frac{1}{a}$ and $\alpha \cdot \beta = a \times \frac{1}{a} = 1$

Here the required equation is $x^2 - \left(a + \frac{1}{a}\right)x + 1 = 0$

Ex.14 The difference of two numbers is 3 and their product is 504. Find the numbers.

Sol. Let the required numbers be x and (x – 3). Then,

$x(x - 3) = 504$

$\Rightarrow x^2 - 3x - 504 = 0$ $\Rightarrow x^2 - 24x + 21x - 504 = 0$

$\Rightarrow x(x-24) + 21(x-24) = 0$ $\Rightarrow (x-24)(x+21) = 0$

$\Rightarrow x - 24 = 0$ or $x + 21 = 0$ $\Rightarrow x = 24$ or $x = -21$

If $x = -21$, then the numbers are –21 and –24.

Again, if $x = 24$, then the numbers are 24 and 21.

Hence, the numbers are – 21, – 24 or 24, 21

Ex.15 The sum of the squares of two consecutive odd positive integers is 290. Find the integers. **[NCERT]**

Sol. Let the two consecutive odd positive integers be x and (x + 2). Then,

$x^2 + (x + 2)^2 = 290$

$\Rightarrow x^2 + x^2 + 4x + 4 = 290$ $\Rightarrow$ $x^2 + 2x - 143 = 0$

$\Rightarrow x^2 + 13x - 11x - 143 = 0$

$\Rightarrow x(x + 13) - 11(x + 13) = 0$

$\Rightarrow (x+13)(x-11) = 0$ $\Rightarrow$ $x = -13$ or $x = 11$

But –13, is not an odd positive integer.

Hence, the required integers are 11 and 13.

Ex.16 Seven years ago Varun's age was five times the square of Swati's age. Three years hence, Swati's age will be two fifth of Varun's age. Find their present ages.

Sol. Let the present ages of Varun and Swati be x years and y years respectively.

Seven years ago,

Varun's age = (x – 7) years and Swati's age = (y – 7) years.

$\therefore \quad (x - 7) = 5 (y - 7)^2$

$\Rightarrow \quad x - 7 = 5 (y^2 - 14 y + 49)$

$\Rightarrow \quad x = 5y^2 - 70y + 245 + 7$

$\Rightarrow \quad x = 5y^2 - 70y + 252 \qquad ...(i)$

Three years hence,

Varun's age = (x + 3) years and Swati's age = (y + 3) years.

$\therefore \quad (y + 3) = \frac{2}{5} (x + 3)$

$\Rightarrow 5y + 15 = 2x + 6 \quad \Rightarrow x = \frac{5y + 9}{2} \qquad ...(ii)$

From (i) and (ii) we get $5y^2 - 70y + 252 = \frac{5y + 9}{2}$

$\Rightarrow \quad 10y^2 - 140y + 504 = 5y + 9$

$\Rightarrow \quad 10y^2 - 145y + 495 = 0 \Rightarrow 2y^2 - 29y + 99 = 0$

$\Rightarrow \quad 2y^2 - 18y - 11y + 99 = 0$

$\Rightarrow \quad 2y (y - 9) - 11 (y - 9) = 0$

$\Rightarrow \quad (y - 9) (2y - 11) = 0 \Rightarrow y = 9 \text{ or } y = \frac{11}{2}$

$\because \quad y = \frac{11}{2}$ is not possible $\qquad \left[\because \frac{11}{2} < 7\right]$

So, y = 9.

$\therefore \quad \frac{5 \times 9 + 9}{2} = 27 \qquad$ [From (ii)]

Hence, the Varun's present age is 27 years and Swati's present age is 9 years.

Ex.17 The length of the hypotenuse of a right triangle exceeds the length of the base by 2 cm and exceeds twice the length of the altitude by 1 cm. Find the length of each side of the triangle.

Sol. Let ΔABC be a right triangle, right angled at B.

Let AB = x. Then

AC = (2x + 1) and

BC = (2x + 1) – 2 = 2x – 1

$\Rightarrow \quad \Delta ABC, AC^2 = AB^2 + BC^2$

[By pythagoras theorem]

$\Rightarrow \quad (2x + 1)^2 = x^2 + (2x - 1)^2$

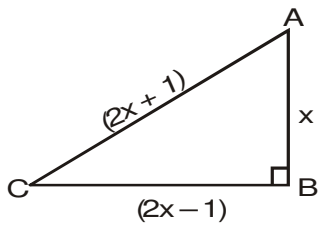
$\Rightarrow \quad 4x^2 + 4x + 1 = x^2 + 4x^2 - 4x + 1$

$\Rightarrow \quad x^2 = 8x \Rightarrow x = 8 \text{ cm}$

$\therefore \quad BC = 2x - 1 = 2 \times 8 - 1 = 15 \text{ cm}$

$AC = 2x + 1 = 2 \times 8 + 1 = 17\text{cm}$

Hence, the sides of the given triangle are 8cm, 15 cm and 17 cm.



Ex.18 Is it possible to design a rectangular park of perimeter 80 cm and area 400 m^2? If so, find its length and breadth. **[NCERT]**

Sol. Let the length and breadth of the rectangular park be ℓ and b respectively. Then,

$2(\ell + b) = 80$

$\ell + b = 40 \Rightarrow \ell = (40 - b)$

And area of the park = 400 m^2

$\therefore \quad \ell b = 400$

$\Rightarrow \quad (40 - b)\, b = 400 \qquad \Rightarrow 40b - b^2 = 400$

$\Rightarrow \quad b^2 - 40b + 400 = 0 \qquad \Rightarrow b^2 - 20b - 20b + 400 = 0$

$\Rightarrow \quad b(b-20) - 20(b-20) = 0 \qquad \Rightarrow (b-20)(b-20) = 0$

$\Rightarrow \quad (b-20)^2 = 0 \qquad \Rightarrow b - 20 = 0 \Rightarrow b = 20$ m

$\therefore \quad \ell = 40 - b = 40 - 20 = 20$ m

Hence, length and breadth of the park are 20 m and 20 m respectively.

Thus, It is possible to design a rectangular park of perimeter 80 m and area 400 m^2

Ex.19 A train travels 360 km at a uniform speed. If the speed had been 5 km/h more, it would have taken 1 hour less for the same journey. Find the speed of the train. **[NCERT]**

Sol. Let the speed of the train be x km/h. Then,

Time taken to cover the distance of 360 km = $\frac{360}{x}$ hours.

If the speed of the train increased by 5 km/h. Then,

Time taken to cover the same distance = $\left(\frac{360}{x+5}\right)$h

According to the question, $\frac{360}{x} - \frac{360}{x+5} = 1$

$\Rightarrow \quad \frac{360(x+5) - 360x}{x(x+5)} = 1$

$\Rightarrow \quad 360x + 1800 - 360x = x^2 + 5x$

$\Rightarrow \quad x^2 + 5x - 1800 = 0 \quad \Rightarrow \quad x^2 + 45x - 40x - 1800 = 0$

$\Rightarrow \quad x(x + 45) - 40(x + 45) = 0$

$\Rightarrow \quad (x+45)(x-40) = 0 \qquad \Rightarrow \qquad x = -45$ or $x = 40$

But the speed can not be negative.

Hence, the speed of the train is 40 km/h.

Ex.20 Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank respectively. Find the time in which each tap can separately fill the tank. **[NCERT]**

Sol. Let the tap of larger diameter takes x hours to fill the tank. Then, the tap of smaller diameter takes (x + 10) hours to fill the tank.

$\therefore$ The portion of tank filled by the larger tap in one hour = $\frac{1}{x}$, the portion of tank filled by the smaller tap in one hour = $\frac{1}{x+10}$

And the portion of tank filled by both the smaller and the larger tap in one hour = $\frac{1}{9\frac{3}{8}} = \frac{8}{75}$

$\therefore \quad \frac{1}{x} + \frac{1}{x+10} = \frac{8}{75}$

$\Rightarrow \quad \frac{x+10+x}{x(x+10)} = \frac{8}{75} \Rightarrow \frac{2x+10}{x^2+10x} = \frac{8}{75}$

$\Rightarrow \quad 150x + 750 = 8x^2 + 80x$

$\Rightarrow \quad 8x^2 - 70x - 750 = 0 \quad \Rightarrow \quad 4x^2 - 35x - 375 = 0$

$\Rightarrow \quad 4x^2 - 60x + 25x - 375 = 0$

$\Rightarrow \quad 4x(x - 15) + 25(x - 15) = 0$

$\Rightarrow \quad (x-15)(4x+25) = 0 \quad \Rightarrow x = 15$ or $x = \frac{-25}{4}$

but the time can not be negative.

Hence, the larger tap takes 15 hours and the smaller tap takes 25 hours.

Ex.21 300 apples are distributed equally among a certain number of students. Had there been 10 more students, each would have received one apple less. Find the number of students.

Sol. Let the number of students be x. Then,

the number of apples received by each student = $\frac{300}{x}$

If there is 10 more students, i.e., (x + 10) students. Then,

the number of apples received by each student = $\frac{300}{x+10}$

According to the question, $\frac{300}{x} - \frac{300}{x+10} = 1$

$\Rightarrow \quad \frac{300x + 3000 - 300x}{x(x+10)} = 1$

$\Rightarrow \quad x^2 + 10x - 3000 = 0 \Rightarrow x^2 + 60x - 50x - 3000 = 0$

$\Rightarrow \quad x(x+60) - 50(x+60) = 0 \quad \Rightarrow \quad (x+60)(x-50) = 0$

$\Rightarrow \quad x = -60$ or $x = 50$

But the number of students can not be negative.

Hence, the number of students is 50.

EXERCISE – I UNSOLVED PROBLEMS

Q.1 Solve : $\frac{x+1}{x-1} + \frac{x-2}{x+2} = 3$.

Q.2 Solve by completion of square method : $2x^2 + 4x - 8 = 0$.

Q.3 Solve using discriminant method : $9x^2 - 12x + 4 = 0$.

Q.4 In the following determine the set of values of 'p' for which the given equation has real roots :
(i) $px^2 + 4x + 1 = 0$
(ii) $2x^2 + px + 3 = 0$

Q.5 The hypotenuse of a right triangle is 25 cm. The difference between the lengths of the other two sides of the triangle is 5 cm. Find the lengths of these sides.

Q.6 Swati can row her boat at a speed of 5 km/h in still water. If it takes her 1 hour more to row the boat 5.25 km upstream then to return downstream, find the speed of the stream.

Q.7 In a fight for 3000 km, an aircraft was slowed down due to bad weather. Its average speed for the trip was reduced by 100 km/hr and consequently time of flight increased by one hour. Find the original duration of flight.

Q.8 Show that :
(i) $x = 3$ is a zero of quadratic polynomial $x^2 - 2x - 3$.
(ii) $x = -2$ is a zero of quadratic polynomial $3x^2 + 7x + 2$.
(iii) $x = 4$ is not a zero of quadratic polynomial $2x^2 - 7x - 5$.

Q.9 In each of the following, determine whether the given values are solutions (roots) of the equation or not :
(i) $3x^2 - 2x - 1 = 0$; $x = 1$
(ii) $x^2 + 6x + 5 = 0$; $x = -1$, $x = -5$
(iii) $x^2 + \sqrt{2}x - 4 = 0$; $x = \sqrt{2}$, $x = -2\sqrt{2}$

Q.10 Solve the following quadratic equations
(i) $x^2 + 5x = 0$ (ii) $x^2 = 3x$
(iii) $x^2 = 4$

Q.11 Solve the following quadratic equations
(i) $7x^2 = 8 - 10x$
(ii) $3(x^2 - 4) = 5x$
(iii) $x(x + 1) + (x + 2)(x + 3) = 42$

Q.12 Solve for x : $12abx^2 - (9a^2 - 8b^2)x - 6ab = 0$

Q.13 If a and c are such that the quadratic equation $ax^2 - 5x + c = 0$ has 10 as the sum of the roots and also as the product of the roots, find a and c.

Q.14 If one of the roots of the quadratic equation $2x^2 + px + 4 = 0$ is 2, find the value of p. also find the value of the other roots.

Q.15 In the following, find the value (s) of p so that the given equation has equal roots.
(i) $3x^2 - 5x + p = 0$
(ii) $2px^2 - 8x + p = 0$

Q.16 Solve :(i) $9x^2 - 3(a + b)x + ab = 0$
(ii) $9x^2 - 3(a^2 + b^2)x + a^2b^2 = 0$

Q.17 Solve :(i) $10x^2 + 3(5a - 2)x - 9 = 0$, $a \neq 0$
(ii) $abx^2 = (a + b)^2 (x - 1)$, $ab \neq 0$

Q.18 Solve :(i) $x^2 - 2ax + (a^2 - b^2) = 0$
(ii) $x^2 - 4ax + 4a^2 - b^2 = 0$

Q.19 Solve :(i) $4x^2 - 2(a^2 + b^2)x + a^2b^2 = 0$
(ii) $9x^2 - 9(a+b)x + (2a^2 + 5ab + 2b^2) = 0$

Q.20 Solve :(i) $x + \frac{1}{x} = 3$, $x \neq 0$
(ii) $x - \frac{1}{x} = 3$, $x \neq 0$

Q.21 Solve :(i) $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}$, $x \neq -4, 7$
(ii) $\frac{1}{x} + \frac{1}{x-2} = 3$, $x \neq 0, 2$.

Q.22 Solve :(i) $\frac{x-1}{x-2} + \frac{x-3}{x-4} = 3\frac{1}{3}$, $x \neq 2, 4$
(ii) $\frac{x}{x+1} + \frac{x+1}{x} = 2\frac{1}{12}$, $x \neq 0, -1$

Q.23 Solve :(i) $2\left(\frac{2x-1}{x+3}\right) - 3\left(\frac{x+3}{2x-1}\right) = 5$, $x \neq -3, \frac{1}{2}$
(ii) $2\left(\frac{2x+3}{x-3}\right) - 25\left(\frac{x-3}{2x+3}\right) = 5$, $x \neq 3, -\frac{3}{2}$

Q.24 (i) Find the vale of k so that the quadratic equation $x^2-2(1+3k)x+7(3+2k)=0$ has equal roots.
(ii) Find the value of c such that the equation $4x^2-2(c+1)x+(c+4)=0$ has real and equal roots.

Q.25 (i) Determine the value(s) of p for which the quadratic equation $2x^2+3x+p=0$ has real roots.
(ii) Determine the value(s) of p for which the quadratic equation $4x^2-3px+9=0$ has real roots.

Q.26 If the equation :

$(1 + m^2)x^2 + 2mcx + (c^2 - a^2) = 0$ has equal roots, prove that $c^2 = a^2(1 + m^2)$.

Q.27 If the roots of the equation

$(a-b)x^2 + (b-c)x + (c-a)=0$ are equal, prove that $2a = b + c$.

Q.28 (i) Find two numbers whose sum is 27 and product is 182.

(ii) The sum of the squares of two natural numbers is 116. If the square of the larger number is 25 times the smaller number, find the numbers.

(iii)The difference of squares of two numbers is 180. The square of the smaller number is 8 times the larger number. Find the two numbers.

(iv) The difference of squares of two natural numbers is 45. The square of the smaller number is four times the larger number. Find the numbers.

Q.29 (i) The difference of two natural numbers is 4 and the difference of their reciprocals is $\frac{1}{8}$,find the numbers.

(ii) The difference of two numbers is 5 and the difference of their reciprocals is $\frac{1}{10}$. Find the numbers.

Q.30 (i) The sum of the numerator and denominator of a certain fraction is 11. If 1 is added to both numerator and denominator, the fraction increases by $\frac{3}{56}$. Find the fraction.

(ii) The numerator of a fraction is one less than its denominator. If three is added to each of the numerator and denominator, the fraction increases by $\frac{3}{28}$. Find the fraction.

(iii)The denominator of a fraction is one more than twice the numerator. If the sum of the fraction and its reciprocal is $2\frac{16}{21}$, find the fraction.

Q.31 (i) A two digit number contains the bigger digit at ten's place. The product of the digits is 27and the difference between the two digits is 6. Find the number.

(ii) A two digit number is seven times the sum of its digits and is also equal to 12 less than three times the product of its digits. Find the number.

Q.32 (i) A rectangular garden 10 m by 16 m is to be surrounded by a concrete walk of uniform width. If the area of the walk is 120 m^2, find the width of the walk.

(ii) Harish made a rectangular garden, with its length 5 metres more than its breadth. Next year, he increased the length by 3 metres and decreased the width by 2 metres. If the area of the garden is 119 sq. m, was this garden larger or smaller ?

(iii) The sum of the areas of two squares is 468 m^2. If the difference of their perimeters is 24 m, find the sides of the two squares.

(iv) The area of a right angled triangle is 600 sq. cm. If the base of the triangle exceeds the altitude by 10 cm, find the dimensions of the triangle.

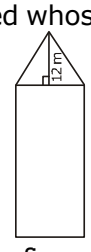
(v)The altitude of a right triangle is 7 cm less than its base. If the hypotenuse is 13 cm, find the other two sides.

(vi)A wire, 112 cm long, is bent to form a right angled triangle. If the hypotenuse is 50 cm long, find the area of the triangle.

(vii) The diagonal of a rectangular field is 60 metres more than the shorter side. If the longer side is 30 metres more than the shorter side, find the sides of the field.

(viii)The length (in cm) of the hypotenuse of a right angled triangle exceeds the length of one side by 2 cm and exceeds twice the length of the other side by 1 cm. Find the length of each side. Also find the perimeter and the area of the triangle.

Q.33 (i) A rectangular park is to be designed whose length is 3 m more than its breadth. Its area is 4 square metres more than the area of a park that has already been made in the shape of an isosceles triangle with its base as the breadth of the rectangular park and of altitude 12 m (shown in the adjoining figure). Find the dimensions of the rectangular park.



(ii) A farmer wishes to grow a 100 m^2 rectangular vegetable garden. Since he has with him only 30 m barbed wire, he fences three sides of the rectangular garden letting compound wall of his house act as the fourth side fence. Find the dimensions of his garden.

(iii) If twice the area of a smaller square is subtracted from the area of a larger square, the result is 14 cm^2. However, if twice the area of the larger square is added to three times the area of the smaller square, the result is 203 cm^2. Determine the sides of the two squares.

Q.34 (i) In a class test, the sum of Shefali's marks in Mathematics and English is 30. Had she got 2 more marks in Mathematics and 3 marks less in English, the product of their marks would have been 210. Find her marks in the two subjects.

(ii) In an auditorium, seats are arranged in rows and columns. The number of rows was equal to the number of seats in each row. When the number of rows was doubled and the number of seats in each row is reduced by 10, the total number of seats increased by 300. Find :

(a) the number of rows in the original arrangement.

(b) the number of seats in the auditorium after re-arrangement.

Q.35 (i) Rohan's mother is 26 years older than him. The product of their ages (in years) 3 years from now will be 360. What is Rohan's present age ?

(ii) Forty years hence Mr. Pratap's age will be the square of what it was 32 years ago. Find his present age.

(iii) The product of Ramu's age (in years) five years ago and his age nine years later is 15. Determine Ramu's present age.

(iv) Is the following situation possible ? If so, determine their present ages. The sum of ages of two friends is 20 years. Four years ago, the product of their ages (in years) was 48.

Q.36 (i) The age of a man is twice the square of the age of his son,. Eight years hence, the age of the man will be 4 years more than three times the age of his son. Find their present ages.

(ii) Two years ago, a man's age was three times the square of his son's age. In three years', his age will be four times his son's age. Find their present ages.

ANSWER KEY

1. $x = -5, 2$ **2.** $x = \sqrt{5} - 1$ or $-\sqrt{5} - 1$

3. $x = \frac{2}{3}$ **4.** **(i)** $p \le 4$ **(ii)** $p \le -2\sqrt{6}$

5. 15 cm, 20 cm **6.** 2 km/hr. **7.** 5 hours

9. **(i)** yes **(ii)** try yourself **(iii)** try yourself

10. **(i)** 0, –5 **(ii)** 0, 3 **(iii)** ± 2

11. **(i)** $x = -2, \frac{4}{7}$ **(ii)** $x = 3,, -\frac{4}{3}$ **(iii)** $x = -6, 3$

12. $x = \frac{3a}{4b}, \frac{2b}{3a}$ **13.** $a = \frac{1}{2}$ and $c = 5$

14. $p = -6$, other root is 1.

15. (i) $p = \frac{25}{12}$ (ii) $p = \pm 2\sqrt{2}$ **16.** (i) $\frac{a}{3}, \frac{b}{3}$ (ii) $\frac{a^2}{3}, \frac{b^2}{3}$

17. (i) $-\frac{3}{2}, \frac{3}{\sqrt{a}}$ (ii) $\frac{a+b}{a}, \frac{a+b}{b}$

18. (i) a + b, a – b (ii) 2a + b, 2a – b

19. (i) $\frac{a^2}{2}, \frac{b^2}{2}$ (ii) $\frac{2a+b}{3}, \frac{a+2b}{3}$

20. (i) $\frac{3 \pm \sqrt{5}}{2}$ (ii) $\frac{3 \pm \sqrt{13}}{2}$ **21.** (i) 1, 2 (ii) $\frac{4 \pm \sqrt{10}}{3}$

22. (i) 5, 5/2 (ii) 3, –4 **23.** (i) $-10, \frac{-1}{5}$ (ii) 6, 1

24. (i) $2, \frac{-10}{9}$ (ii) –3, 5 **25.** (i) $p \le \frac{9}{8}$

(ii) $p \ge 4$ or $p \le -4$

28. (i) 13, 14 (ii) 4, 10
(iii) 18, 12, or 18, –12 (iv) 9, 6

29. (i) 8, 4 (ii) 10, 5 or –5, –10

30. (i) $\frac{4}{7}$ (ii) $\frac{3}{4}$ (iii) $\frac{3}{7}$

31. (i) 93 (ii) 84

32. (i) 2m (ii) 7 sq. m
(iii) 18m, 12m
(iv) 30cm, 40cm, 50cm
(v)12cm,5cm (vi) $336cm^2$
(vii) 120m, 90m
(viii) 15cm, 8cm, 17cm; 40cm, $60cm^2$

33. (i) 7m, 4m
(ii) 20m×5m or 10m ×10m
(iii) 5cm, 8cm

34. (i) 12, 18 or 13, 17
(ii) (a) 30 (b) 1200

35. (i) 7 years (ii) 41 years (iii) 6 years
(iv) No

36. (i) 32 years 4 years (ii) 29 years, 5 years

EXERCISE – II BOARD PROBLEMS

Q.1 Find the values of k so that (x – 1) is a factor of $k^2x^2 - 2kx - 3$. **[CBSE-Delhi-2003]**

Q.2 Solve using the quadratic formula :

$x^2 - 4x + 1 = 0$ **[ICSE-2003]**

Q.3 Solve for x: $4x^2 - 2(a^2 + b^2)x + a^2b^2 = 0$

[CBSE-Delhi-2004]

Q.4 Solve for x : $4x^2 - 4a^2x + (a^4 - b^4) = 0$

[CBSE-Delhi-2004]

Q.5 Solve for x : $9x^2-9(a+b)x+[2a^2+5ab+2b^2]=0$

[CBSE-Delhi-2004]

Q.6 Using quadratic formula, solve the following quadratic equation for x : $p^2x^2 + (p^2 - q^2)x - q^2 = 0$ **[CBSE-AI-2004]**

Q.7 Using quadratic formula, solve the following quadratic equation for x : $x^2 - 2ax + (a^2 - b^2) = 0$ **[CBSE-AI-2004]**

Q.8 Using quadratic formula, solve the following quadratic equation for x : $x^2 - 4ax + 4a^2 - b^2 = 0$. **[CBSE-AI-2004]**

Q.9 Solve for x : $9x^2 - 6a^2x + (a^4 - b^4) = 0$

[CBSE-Foreign-2004]

Q.10 Solve for x : $9x^2 - 6ax + (a^2 - b^2) = 0$.

[CBSE-Foreign-2004]

Q.11 Solve for x : $16x^2 - 8a^2x + (a^4 - b^4) = 0$

[CBSE-Foreign-2004]

Q.12 Solve for x : $36x^2 - 12ax + (a^2 - b^2) = 0$.

[CBSE-Delhi-2004C]

Q.13 Solve the equation $3x^2 - x - 7 = 0$ and give your answer correct to two decimal places.

[ICSE-2004]

Q.14 Solve for x : $4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0$

[CBSE-Foreign-2005]

OR

Solve for x : $x^2 - 2(a^2 + b^2)x + (a^2 - b^2)^2 = 0$ **[CBSE-Delhi-2006C]**

Q.15 Solve $x^2 - 5x - 10 = 0$ and give your answer correct to two decimal places **[ICSE-2005]**

Q.16 Using quadratic formula, solve for x : $9x^2 - 3(a + b)x + ab = 0$

OR

The sum of the squares of two consecutive natural numbers is 421. Find the numbers.

[CBSE-Delhi-2005C]

Q.17 Using quadratic formula, solve the following for x : $9x^2 - 3(a^2 + b^2)x + a^2b^2 = 0$

OR

The sum of the squares of three consecutive positive integers is 50. Find the integers.

[CBSE-AI-2005C]

Q.18 Rewrite the following as a quadratic equation in x and then solve for x : $\frac{4}{x} - 3 = \frac{5}{2x+3}, x \neq 0, -\frac{3}{2}$ **[CBSE-AI-2006C]**

Q.19 Solve $2x - \frac{1}{x} = 7$ and give your answer correct to 2 decimal places. **[ICSE-2006]**

Q.20 Solve $x^2 - 3x - 9 = 0$ and give your answer correct to 2 decimal places. **[ICSE-2007]**

Q.21 Find the roots of the following equation : $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}$; $x \neq -4, 7$

[CBSE-Delhi-2008]

Q.22 Is x = – 2 a solution of the equation $x^2 - 2x + 8 = 0$? **[CBSE-AI-2008]**

Q.23 Is x = – 3 a solution of the equation $2x^2 + 5x + 3 = 0$? **[CBSE-AI-2008]**

Q.24 Is x = – 4 a solution of the equation $2x^2$ + 5x – 12 = 0? **[CBSE-AI-2008]**

Q.25 Show that x = – 3 is a solution of x^2 + 6x + 9 = 0. **[CBSE-Foreign-2008]**

Q.26 Show that x = – 3 is a solution of $2x^2$ + 5x – 3 = 0. **[CBSE-Foreign-2008]**

Q.27 Show that x = – 2 is a solution of $3x^2$ + 13x + 14 = 0. **[CBSE-Foreign-2008]**

Q.28 Find the discriminant of the quadratic equation $3\sqrt{3}x^2 + 10x + \sqrt{3} = 0$. **[CBSE-AI-2009]**

Q.29 The sum of two numbers is 8. Determine the numbers if the sum of their reciprocals is $\frac{8}{15}$.

[CBSE-AI-2009]

Q.30 Write the nature of roots of quadratic equation $4x^2 + 4\sqrt{3}x + 3 = 0$. **[CBSE-Foreign-2009]**

Q.31 An aeroplane travelled a distance of 400 km at an average speed of x km/hr. On the return journey, the speed was increased by 40 km/hr. Write down an expression for the time taken for (i) the onward journey, (ii) the return journey. If the return journey took 30 minutes less than the onward journey, write an equation in x and find the value of x. **[ICSE-2002]**

Q.32 In an auditorium, seats were arranged in rows and columns. The number of rows was equal to number of seats in each row. When the number of rows was doubled and the number of seats in each row was reduced by 10, the total number of seats increased by 300. Find (i) the number of rows in the original arrangement, (ii) the number of seats in the auditorium after rearrangement. **[ICSE-2003]**

Q.33 Solve for x : $2\left(\frac{2x-1}{x+3}\right) - 3\left(\frac{x+3}{2x-1}\right) = 5$; given that $x \neq -3$, $x \neq \frac{1}{2}$. **[CBSE-Delhi-2004]**

Q.34 Solve for x : $2\left(\frac{x-1}{x+3}\right) - 7\left(\frac{x+3}{x-1}\right) = 5$; given that $x \neq -3$, $x \neq 1$. **[CBSE-Delhi-2004]**

Q.35 Solve for x : $2\left(\frac{2x+3}{x-3}\right) - 25\left(\frac{x-3}{2x+3}\right) = 5$; given that $x \neq 3$, $x \neq \frac{-3}{2}$. **[CBSE-Delhi-2004]**

Q.36 Solve for x : $\left(\frac{4x-3}{2x+1}\right) - 10\left(\frac{2x+1}{4x-3}\right) = 3$; given that $x \neq \frac{-1}{2}; x \neq \frac{3}{4}$.

OR

300 apples are distributed equally among a certain number of students. Had there been 10 more students, each would have received one apple less. Find the number of students. **[CBSE-AI-2004]**

Q.37 Solve for x : $2\left(\frac{x+2}{2x-3}\right) - 9\left(\frac{2x-3}{x+2}\right) = 3$; given that $x \neq \frac{3}{2}$; $x \neq -2$.

OR

An aeroplane takes one hour less for a journey of 1200 km if its speed is increased by 100 km/hour from its usual speed. Find its usual speed. **[CBSE-Foreign-2004]**

Q.38 A two digit number is four times the sum of its digits and is also equal to twice the product of its digits. Find the number. **[CBSE-Delhi-2004C]**

Q.39 A two digit number is seven times the sum of its digits and is also equal to 12 less than three times the product of its digits. Find the number **[CBSE-Delhi-2004C]**

Q.40 A two digit number is 5 times the sum of its digits and is also equal to 5 more than twice the product of its digits. Find the number. **[CBSE-Delhi-2004C]**

Q.41 The sum of two numbers a and b is 15, and the sum of their reciprocals $\frac{1}{a}$ and $\frac{1}{b}$ is $\frac{3}{10}$. Find the number a and b. **[CBSE-Delhi-2005]**

Q.42 The sum of two numbers is 16. The sum of their reciprocals is $\frac{1}{3}$. Find the numbers. **[CBSE-Delhi-2005]**

Q.43 The sum of two numbers is 18. The sum of their reciprocals is $\frac{1}{4}$. Find the numbers. **[CBSE-Delhi-2005]**

Q.44 A two digit number is such that the product of its digits is 15. If 18 is added to the number, the digits interchange their places. Find the number. **[CBSE-AI-2005]**

Q.45 A two digit number is such that the product of its digits is 20. If 9 is added to the number, the digits interchange their places. Find the number. **[CBSE-AI-2005]**

Q.46 A two digit number is such that the product of its digits is 14. If 45 is added to the number, the digits interchange their places. Find the number. **[CBSE-AI-2005]**

Q.47 The sum of the squares of two natural numbers is 34. If the first number is one less than twice the second number, find the numbers. **[CBSE-Foreign-2005]**

Q.48 A passenger train takes 2 hours less for a journey of 300 km if its speed is increased by 5 km/hour from its usual speed. Find the usual speed of the train. **[CBSE-Delhi-2005C, 2006]**

Q.49 Solve for x $\frac{x+1}{x-1}+\frac{x-2}{x+2}=3$: $(x \neq 1, -2)$ **[CBSE-AI-2005C]**

OR

Aeroplane left 30 minutes later than its scheduled time and in order to reach destination 1500 km away in time, it has to increase its speed by 250 km/h from its usual speed. Determine its usual speed.

Q.50 Solve for x : $\frac{1}{a+b+x}=\frac{1}{a}+\frac{1}{b}+\frac{1}{x}$; $a \neq 0, b \neq 0, x \neq 0$

OR

Solve for x : $abx^2 + (b^2 - ac)x - bc = 0$ **[CBSE-Delhi-2005]**

Q.51 Solve for x : $a^2b^2x^2 + b^2x - a^2x - 1 = 0$

OR

Solve for x : $\frac{x-1}{x-2}+\frac{x-3}{x-4}=3\frac{1}{3}$ $(x \neq 2, 4)$ **[CBSE-AI-2005]**

Q.52 By increasing the speed of a car by 10 km/hr, the time of journey for a distance of 72 km is reduced by 36 minutes. Find the original speed of the car. **[ICSE-2005]**

Q.53 Solve for x : $12abx^2 - (9a^2 - 8b^2)x - 6ab = 0$

OR

A two digit number is such that the product of its digits is 35. When 18 is added to number, the digits interchange their places. Find the number. **[CBSE-Delhi-2006]**

Q.54 Using the quadratic formula, solve the equation : $a^2b^2x^2-(4b^4-3a^4)\,x-12a^2b^2=0$

OR

The sum of two natural numbers is 8. Determine the number if the sum of their reciprocals is $\frac{8}{15}$. **[CBSE-AI-2006]**

Q.55 Solve for x : $(a + b)^2 x^2 + 8(a^2 - b^2)x + 16(a - b)^2 = 0$

OR

Two numbers differ by 3 and their product is 504. Find the numbers. **[CBSE-Foreign-2006]**

Q.56 A fast train takes 3 hours less than a slow train for a journey of 600 km. If the speed of the slow train is 10 km/hr less than that of the fast train, find the speeds of the two trains. **[CBSE-Foreign-2006]**

Q.57 Seven years ago Varun's age was five times the square of Swati's age. Three years hence Swati's age will be two-fifth of Varun's age. Find their present ages. **[CBSE-Delhi-2006C]**

Q.58 A 2-digit number is such that product of its digits is 18. When 63 is subtracted from the number, the digits interchange their places. Find the number.

OR

A train covers a distance of 90 km at a uniform speed. Had the speed been 15 km/hour more, it would have taken 30 minutes less for the journey. Find the original speed of the train **[CBSE-AI-2006C]**

Q.59 A shopkeeper buys x books for Rs. 720. (i) Write the cost of 1 book in terms of x, (ii) If the cost per book were Rs. 5 less, the number of books that could be bought for Rs. 720 would be 2 more.

Write down the equation in x for the above situation and solve it to find x. **[ICSE-2006]**

Q.60 The difference of two numbers is 5 and the difference of their reciprocals is $\frac{1}{10}$. Find the numbers.

OR

By increasing the list price of a book by Rs. 10 a person can buy 10 less books for Rs. 1200. Find the original list price of the book. **[CBSE-Delhi-2007]**

Q.61 The numerator of a fraction is one less than its denominator. If three is added to each of the numerator and denominator, the fraction is increased by $\frac{3}{28}$. Find the fraction.

OR

The difference of squares of two natural numbers is 45. The square of the smaller number is four times the larger number. Find the numbers. **[CBSE-AI-2007]**

Q.62 Some students planned a picnic. The budget for the food was Rs. 480. As eight of them failed to join the party, the cost of the food for each member increased by Rs. 10. Find how many students went for the picnic. **[ICSE-2008]**

Q.63 In a class test, the sum of the marks obtained by P in mathematics and science is 28. Had he got 3 more marks in mathematics and 4 marks less in science, the product of marks obtained in the two subjects would have been 180. Find the marks obtained in the two subjects separately.

OR

The sum of the areas of two squares is 640 m^2. If the difference in their perimeters be 64 m, find the sides of the two squares. **[CBSE-Delhi-2008]**

Q.64 A motor boat whose speed is 18 km/h in still water takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.

OR

Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank. **[CBSE-AI-2008]**

Q.65 A peacock is sitting on the top of a pillar, which is 9 m high. From a point 27 m away from the bottom of the pillar, a snake is coming to it's hole at the base of the pillar. Seeing the snake the peacock pounces on it. If their speeds are equal, at what distance from the hole is the snake caught?

OR

The difference of two numbers is 4. If the difference of their reciprocals is $\frac{4}{21}$, find the two numbers. **[CBSE-Foreign-2008]**

Q.66 The sum of the squares of two consecutive odd numbers is 394. Find the numbers. **[CBSE-Delhi-2009]**

Q.67 Solve the following equation for x : $9x^2 - 9(a + b)x + (2a^2 + 5ab + 2b^2) = 0$.

OR

If (-5) is a root of the quadratic equation $2x^2 + px - 15 = 0$ and the quadratic equation $p(x^2 + x) + k = 0$ has equal roots, then find the values of p and k. **[CBSE-AI-2009]**

Q.68 A trader bought a number of articles for Rs. 900. Five articles were found damaged. He sold each of the remaining articles at Rs. 2 more than what he paid for it. He got a profit of Rs. 80 on the whole transaction. Find the number of articles he bought.

OR

Two years ago a man's age was three times the square of his son's age. Three years hence his age will be four times his son's age. Find their present ages. **[CBSE-Foreign-2009]**

Q.69 A girl is twice as old as her sister. Four years hence, the product of their ages (in years) will be 160. Find their present ages. **[CBSE-AI-2010]**

ANSWER KEY

1. $(-1, 3)$ **2.** $[2+\sqrt{3}, 2-\sqrt{3}]$

3. $x = \frac{a^2}{2}, \frac{b^2}{2}$

4. $x = \frac{(a^2+b^2)}{2}, \frac{(a^2-b^2)}{2}$

5. $x = \frac{(2a+b)}{3}, \frac{(a+2b)}{3}$

6. $\frac{q^2}{p^2}, -1$ **7.** a + b, a − b

8. 2a + b, 2a − b **9.** $\frac{(a^2+b^2)}{3}, \frac{(a^2-b^2)}{3}$

10. $\frac{(a+b)}{3}, \frac{(a-b)}{3}$ **11.** $\frac{(a^2+b^2)}{4}, \frac{(a^2-b^2)}{4}$

12. $\frac{(a+b)}{6}, \frac{(a-b)}{6}$ **13.** 1.70, − 1.37

14. $\frac{\sqrt{3}}{4}, \frac{-2}{\sqrt{3}}$ or $(a + b)^2, (a - b)^2$

15. 6.53, − 1.53 **16.** $\frac{a}{3}, \frac{b}{3}$ or 14, 15

17. $\frac{a^2}{3}, \frac{b^2}{3}$ or 3, 4, 5 **18.** x = − 2, 1

19. 3.64, −0.14 **20.** 4.85, −1.85

21. 2, 1 **22.** No **23.** No

24. Yes **28.** 64 **29.** 3 and 5

30. Real and equal roots

31. (i) $\left(\frac{400}{x}\right)$ hrs. (ii) $\left(\frac{400}{x+40}\right)$ hrs; x = 160 km/hr

32. (i) 30 (ii) 1200 **33.** x = − 10, − 1/5

34. $x = -\frac{23}{5}, -1$ **35.** x = 6,1

36. $x = -\frac{4}{3}, \frac{1}{8}$ or 50

37. $x = \frac{5}{8}, \frac{11}{5}$ or 300 km / hr **38.** 36

39. 84 **40.** 45 **41.** 5,10

42. 4,12 **43.** 6,12 **44.** 35

45. 45 **46.** 27 **47.** 5 and 3

48. 25 km/hr

49. x = − 5, 2 or 750 km/hr

50. x = − a, − b or $x = \frac{c}{b}, \frac{-b}{a}$

51. $x = \frac{1}{b^2}, -\frac{1}{a^2}$ or $x = \frac{5}{2}, 5$ **52.** 30 km/hr

53. $x = \frac{-2b}{3a}, \frac{3a}{4b}$ or 57

54. $x = \frac{-3a^2}{b^2}, \frac{4b^2}{a^2}$ or 3 and 5

55. $x = \frac{-4(a-b)}{a+b}$ or 21, 24 or − 21, − 24

56. 40 km/hr, 50 km/hr **57.** 9 years, 27 years

58. 92 or 45 km/hr

59. (i) Rs. $\left(\frac{720}{x}\right)$ (ii) $x^2 + 2x - 288 = 0$, x = 16

60. 10 and 5 or Rs. 30 **61.** 3/4 or 9 and 6

62. 16

63. Marks in maths : 12(9), Marks in science : 16 (19)
or
Sides of two square are 8m & 24m

64. 6 km/hr or 25 hrs and 15 hrs

65. 12 m or (7 and 3) or (−3 and −7)

66. 13 and 15

67. $\frac{2a+b}{3}, \frac{a+2b}{3}$ or p = 7 and $k = \frac{7}{4}$

68. 75 or son's age = 5 years and man's age = 29 years.

69. 6 years and 12 years

EXERCISE – III MULTIPLE CHOICE QUESTIONS

Q.1 The roots of the equation $3x^2 - 5x + 2 = 0$

(A) $\frac{3}{2}, 1$ (B) $-1, \frac{2}{3}$

(C) $\frac{2}{3}, 1$ (D) $-\frac{2}{3}, 1$

Q.2 The roots of the equation
$x^2 + x - (a + 1)(a + 2) = 0$ are
(A) $(a + 1), (a + 2)$
(B) $-(a + 1), -(a + 2)$
(C) $-(a + 1), (a + 2)$
(D) $(a + 1), (-a - 2)$

Q.3 The roots of the equation
$a^2b^2x^2 + (b^2 - a^2)x - 1 = 0$ are

(A) $\frac{1}{a^2}, \frac{1}{b^2}$ (B) $-\frac{1}{a^2}, \frac{1}{b^2}$

(C) $-\frac{1}{a^2}, -\frac{1}{b^2}$ (D) $\frac{1}{a^2}, -\frac{1}{b^2}$

Q.4 The roots of the equation
$4x^2 + 4bx - a^2 + b^2 = 0$ are

(A) $(a + b), (a - b)$ (B) $\left(\frac{a+b}{2}\right), \left(\frac{a-b}{2}\right)$

(C) $-\left(\frac{a+b}{2}\right), \left(\frac{a-b}{2}\right)$ (D) $-\left(\frac{a+b}{2}\right), -\left(\frac{a-b}{2}\right)$

Q.5 The roots of the equation $x^2 - \sqrt{3}x - x + \sqrt{3} = 0$ are

(A) $\sqrt{3}, 1$ (B) $\sqrt{3}, -1$

(C) $-\sqrt{3}, -1$ (D) $-\sqrt{3}, 1$

Q.6 The roots of the equation $2x^2 + 5\sqrt{3}x + 6 = 0$ are

(A) $\frac{\sqrt{3}}{2}, 2\sqrt{3}$ (B) $-\frac{\sqrt{3}}{2}, -2\sqrt{3}$

(C) $\frac{\sqrt{3}}{2}, -2\sqrt{3}$ (D) $-\frac{\sqrt{3}}{2}, 2\sqrt{3}$

Q.7 The roots of the equation
$ab\ x^2 + b^2\ x - acx - bc = 0$ are

(A) $\frac{c}{b}, -\frac{b}{c}$ (B) $\frac{c}{a}, \frac{b}{c}$

(C) $\frac{c}{b}, -\frac{b}{a}$ (D) $-\frac{c}{b}, \frac{b}{a}$

Q.8 The value(s) of k for which the equation $9x^2 + 3kx + 4 = 0$ has real and equal roots is (are)
(A) 4 (B) –4

(C) $4, \frac{1}{4}$ (D) 4, –4

Q.9 The value(s) of k for which the equation $kx^2 - 5x + k = 0$ has real and equal roots is (are)

(A) $\frac{5}{2}$ (B) $\frac{5}{2}, \frac{-5}{2}$

(C) $\frac{-5}{2}$ (D) 10

Q.10 The value of k for which the equation $(k + 1)x^2 - 2kx + 2x + 1 = 0$ has real and equal roots are

(A) 2, 1 (B) $0, \frac{3}{4}$

(C) 0, 1 (D) 0, 3

Q.11 If the equation $x^2 + 4x + k = 0$ has real and distinct roots, then
(A) $k < 4$ (B) $k > 4$
(C) $k \geq 4$ (D) $k \leq 4$

Q.12 If the equation $x^2 - mx + 1 = 0$ has two distinct roots, then
(A) $|m| = 2$ (B) $|m| > 2$
(C) $|m| < 2$ (D) $m > -2$

Q.13 If the equation $mx^2 + 2x + m = 0$ has two equal roots, then
(A) $m = \pm 1$ (B) $m = 0$
(C) $m = 0, 1$ (D) $m = -1, 0$

Q.14 If the equation $x^2 - ax + 1 = 0$ has no real root, then
(A) $a > 2$ (B) $a < -2$
(C) $-3 < a < 3$ (D) $-2 < a < 2$

Q.15 If the roots of the equation $(a^2 + b^2)x^2 - 2b(a + c)x + (b^2 + c^2) = 0$ are equal, then
(A) $b^2 = ac$ (B) $b = ac$

(C) $b = \frac{2ac}{b+c}$ (D) $2b = a + c$

Q.16 The value of k for which both the equations $x^2 + kx + 64 = 0$ and $x^2 - 8x + k = 0$ have equal roots, is
(A) 4 (B) 8
(C) 12 (D) 16

Q.17 If the equation $(m^2 + n^2)x^2 - 2(mp + nq)x + p^2 + q^2 = 0$ has equal roots, then

(A) mn = pq (B) $mn = \sqrt{pq}$

(C) mq = np (D) $mq = \sqrt{np}$

Q.18 If p and q are the roots of the equation $x^2 + px - q = 0$, then

(A) p = −2, q = 0 (B) p = 0, q = 1

(C) p = 1, q = −2 (D) p = -2, q = 1

Q.19 Which of the following is a quadratic equation?

(A) $x^2 - 3\sqrt{x} + 2 = 0$ (B) $x + \frac{1}{x} = x^2$

(C) $x^2 + \frac{1}{x^2} = 5$ (D) $2x^2 - 5x = (x - 1)^2$

Q.20 Which of the following is a quadratic equation?

(A) $(x^2 + 1) = (2 - x)^2 + 3$

(B) $x^3 - x^2 = (x - 1)^3$

(C) $2x^2 + 3 = (5 + x)(2x - 3)$

(D) None of these

Q.21 Which of the followings is not a quadratic equation

(A) $3x - x^2 = x^2 + 5$

(B) $(x + 2)^2 = 2(x^2 - 5)$

(C) $(\sqrt{2}x + 3)^2 = 2x^2 + 6$

(D) $(x - 1)^2 = 3x^2 + x - 2$

Q.22 If x = 3 is a solution of the equation $3x^2 + (k - 1)x + 9 = 0$, then k = ?

(A) 11 (B) −11

(C) 13 (D) −13

Q.23 The sum of the roots of the equation $x^2 - 6x + 2 = 0$ is

(A) 2 (B) −2

(C) 6 (D) −6

Q.24 If the product of the roots of the equation $x^2 - 3x + k = 10$ is −2, then the value of k is

(A) −2 (B) −8

(C) 8 (D) 12

Q.25 If one root of the equation $2x^2 + ax + 6 = 0$ is 2, then a = ?

(A) 7 (B) −7

(C) $\frac{7}{2}$ (D) $\frac{-7}{2}$

Q.26 The ratio of the sum and product of the roots of the equation $7x^2 - 12x + 18 = 0$ is

(A) 7 : 12 (B) 7 : 18

(C) 2 : 3 (D) 3 : 2

Q.27 The roots of the equation $4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0$ are

(A) $\frac{2\sqrt{3}}{3}, \frac{-\sqrt{3}}{4}$ (B) $\frac{-2\sqrt{3}}{3}, \frac{\sqrt{3}}{4}$

(C) $\frac{\sqrt{3}}{3}, \frac{-\sqrt{3}}{4}$ (D) $\frac{-\sqrt{3}}{3}, \frac{\sqrt{3}}{4}$

Q.28 The roots of the equation $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$ are

(A) $\sqrt{2}, \frac{5\sqrt{2}}{2}$ (B) $-\sqrt{2}, \frac{5\sqrt{2}}{2}$

(C) $\sqrt{2}, \frac{-5\sqrt{2}}{2}$ (D) $-\sqrt{2}, \frac{-5\sqrt{2}}{2}$

Q.29 The roots of the equation $3^{x+2} + 3^{-x} = 10$ are

(A) 2, 0 (B) −2, 0

(C) 3, −1 (D) −3, 1

Q.30 The roots of the equation $3x^2 - 2\sqrt{6}x + 2 = 0$ are

(A) $\sqrt{\frac{3}{2}}, \sqrt{\frac{3}{2}}$ (B) $\sqrt{\frac{2}{3}}, \sqrt{\frac{2}{3}}$

(C) $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$ (D) $\frac{2}{\sqrt{3}}, \frac{2}{\sqrt{3}}$

Q.31 The roots of the equation $\frac{x}{x+1} + \frac{x+1}{x} = \frac{34}{15}$ are

(A) $\frac{5}{2}, \frac{3}{2}$ (B) $\frac{5}{2}, \frac{-3}{2}$

(C) $\frac{-5}{2}, \frac{3}{2}$ (D) $\frac{-5}{2}, \frac{-3}{2}$

Q.32 The roots of the equation $\sqrt{\frac{x}{1-x}} + \sqrt{\frac{1-x}{x}} = 2\frac{1}{6}$ are

(A) $\frac{9}{13}, \frac{7}{13}$ (B) $\frac{9}{13}, \frac{4}{13}$

(C) $\frac{7}{13}, \frac{4}{13}$ (D) None of these

Q.33 The roots of $\frac{x+4}{x-4} + \frac{x-4}{x+4} = \frac{10}{3}$ are

(A) ±4 (B) ±6

(C) ±8 (D) $2 \pm \sqrt{3}$

Q.34 The root of a quadratic equation are 5 and –2. Then, the equation is
(A) $x^2 - 3x + 10 = 0$ (B) $x^3 - 3x - 10 = 0$
(C) $x^2 + 3x - 10 = 0$ (D) $x^2 + 3x + 10 = 0$

Q.35 If the sum of the roots of a quadratic equation is 6 and their product is 6, the equation is
(A) $x^2 - 6x + 6 = 0$ (B) $x^2 + 6x - 6 = 0$
(C) $x^2 - 6x - 6 = 0$ (D) $x^2 + 6x + 6 = 0$

Q.36 If one root of the equation $3x^2 - 10x + 3 = 0$ is $\frac{1}{3}$, then the other root is
(A) $\frac{-1}{3}$ (B) $\frac{1}{3}$
(C) –3 (D) 3

Q.37 The quadratic equation whose one root is $(3 + 2\sqrt{3})$ is
(A) $x^2 + 6x - 3 = 0$ (B) $x^2 - 6x - 3 = 0$
(C) $x^2 + 6x + 3 = 0$ (D) $x^2 - 6x + 3 = 0$

Q.38 If the sum of the roots of the equation $kx^2 + 2x + 3k = 0$ is equal to their product, then the value of k is
(A) $\frac{1}{3}$ (B) $\frac{-1}{3}$
(C) $\frac{2}{3}$ (D) $\frac{-2}{3}$

Q.39 If one root of $5x^2 + 13x + k = 0$ be the reciprocal of the other root, then the value of k is
(A) 0 (B) 1
(C) 2 (D) 5

Q.40 The roots of the equation $ax^2 + bx + c = 0$ will be reciprocal of each other if
(A) a = b (B) b = c
(C) c = a (D) none of these

Q.41 If the roots of the equation $ax^2 + bx + c = 0$ are equal, then c = ?
(A) $\frac{-b}{2a}$ (B) $\frac{b}{2a}$
(C) $\frac{-b^2}{4a}$ (D) $\frac{b^2}{4a}$

Q.42 If the equation $9x^2 + 6kx + 4 = 0$ has equal roots, then k = ?
(A) 2 or 0 (B) –2 or 0
(C) 2 or –2 (D) 0 only

Q.43 If the equation $x^2 + 2(k + 2)x + 9k = 0$ has equal roots, then k = ?
(A) 1 or 4 (B) –1 or 4
(C) 1 or –4 (D) –1 or –4

Q.44 If the equation $4x^2 - 3kx + 1 = 0$ has equal roots, then k = ?
(A) $\pm\frac{2}{3}$ (B) $\pm\frac{1}{3}$
(C) $\pm\frac{3}{4}$ (D) $\pm\frac{4}{3}$

Q.45 If the equation $x^2 - 2x(1 + 3k) + 7(3 + 2k) = 0$ has equal roots, then k = ?
(A) 2 or $\frac{10}{9}$ (B) –2 or $\frac{10}{9}$
(C) 2 or $\frac{-10}{9}$ (D) –2 or $\frac{-10}{9}$

Q.46 If the equation $(a^2 + b^2)x^2 - 2b(a + c)x + (b^2 + c^2) = 0$ has both roots equal, then
(A) b = ac (B) $b = \frac{1}{2}(a + c)$
(C) $b^2 = ac$ (D) $b = \frac{2ac}{(a + c)}$

Q.47 The roots of $ax^2 + bx + c = 0$, $a \neq 0$ are real and unequal, if $(b^2 - 4ac)$
(A) > 0 (B) = 0
(C) < 0 (D) None of these

Q.48 In the equation $ax^2 + bx + c = 0$, it is given that $D = (b^2 - 4ac) > 0$. Then, the roots of the equation are
(A) real and equal (B) real and unequal
(C) imaginary (D) None of these

Q.49 If the equation $x^2 + 5kx + 16 = 0$ has no real roots, then
(A) $k > \frac{8}{5}$ (B) $k < \frac{-8}{5}$
(C) $\frac{-8}{5} < k < \frac{8}{5}$ (D) None of these

Q.50 If the equation $x^2 - kx + 1 = 0$ has no real roots, then
(A) k < –2 (B) k > 2
(C) –2 < k < 2 (D) None of these

Q.51 The roots of the equation $2x^2 - 6x + 7 = 0$ are
(A) real, unequal and rational
(B) real, unequal and irrational
(C) real and equal
(D) imaginary

Q.52 The roots of the equation $2x^2 - 6x + 3 = 0$ are
(A) real, unequal and rational
(B) real, unequal and irrational
(C) real and equal
(D) imaginary

Q.53 For the equation $ax^2 + bx + c = 0$, which of the following statements is incorrect ?
(A) If $(b^2 - 4ac) < 0$, the roots are imaginary
(B) If $(b^2 - 4ac) = 0$, the roots are real and equal
(C) If $(b^2 - 4ac) > 0$ and $(b^2 - 4ac)$ is a perfect square, then the roots are rational and unequal.
(D) $(b^2 - ac) < 0$, the roots are irrational

Q.54 If the roots of $5x^2 - kx + 1 = 0$ are real and distinct, then
(A) $-2\sqrt{5} < k < 2\sqrt{5}$
(B) $k > 2\sqrt{5}$ only
(C) $k > -2\sqrt{5}$ only
(D) either $k > 2\sqrt{5}$ or $k < -2\sqrt{5}$

Q.55 The roots of the equation $3x^2 + 7x + 8 = 0$ are
(A) both real and equal
(B) both real and unequal
(C) both imaginary
(D) none of these

Q.56 The sum of a number and its reciprocal is $2\frac{1}{20}$. The number is
(A) $\frac{5}{4}$ (B) $\frac{4}{3}$
(C) $\frac{3}{4}$ (D) $\frac{1}{6}$

Q.57 The two parts into which 57 should be divided so that their product is 782, are
(A) 43 and 14 (B) 33 and 24
(C) 34 and 23 (D) 44 and 13

Q.58 The perimeter of a rectangle is 82 m and its area is 400 m^2. The breadth of the rectangle is
(A) 25 m (B) 20 m
(C) 16 m (D) 9 m

Q.59 Which constant should be added and subtracted to solve the quadratic equation $4x^2 - \sqrt{3}x - 5 = 0$ by the method of completing the square ?
(A) $\frac{9}{10}$ (B) $\frac{3}{16}$
(C) $\frac{3}{4}$ (D) $\frac{\sqrt{3}}{4}$

Q.60 The roots of the equation $2x - \frac{3}{x} = 1$ are
(A) $\frac{1}{2}, -1$ (B) $\frac{3}{2}, 1$
(C) $\frac{3}{2}, -1$ (D) $\frac{-1}{2}, \frac{3}{2}$

Q.61 For what real values of k, the equation $9x^2 + 8kx + 16 = 0$ has real and equal roots ?
(A) k = 2 or −2 (B) k = 3 or −3
(C) $k = \frac{4}{3}$ or $\frac{-4}{3}$ (D) None of these

Q.62 For what values of k, the equation $kx^2 - 6x - 2 = 0$ has real roots ?
(A) $k \le \frac{-9}{2}$ (B) $k \ge \frac{-9}{2}$
(C) $k \le -2$ (D) None of these

Q.63 If α and β are the roots of the equation $3x^2 + 8x + 2 = 0$, then $\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)$ = ?
(A) $\frac{-3}{8}$ (B) $\frac{2}{3}$
(C) −4 (D) 4

Q.64 The value of the expression $16x^2 + 24x + 9$ for $x = -\frac{3}{4}$ is -
(A) 2 (B) 1
(C) 0 (D) −1

Q.65 x = 3 is a solution of the equation $3x^2 + (k - 1)x + 9 = 0$ if k has value
(A) 13 (B) −13
(C) 11 (D) −11

Q.66 The roots of the equation $x^2 + 2x - 35 = 0$ are -
(A) 5, 7 (B) −5, −7
(C) −5, 7 (D) 5, − 7

Q.67 Match List I with List II :

List I	**List II**
a. Roots of $2x^2 - 9x + 7 = 0$	$\frac{7}{2}$ and 7
b. Roots of $2x^2 - 21x + 49 = 0$	$\frac{7}{2}$ and 1
c. Roots of $x^2 - 6x + 9 = 0$	$\frac{7}{2}$ and 3
d. Roots of $2x^2 - 13x + 21 = 0$	3 and 3

a b c d	a b c d
(A) 2 1 4 3	(B) 1 2 3 4
(C) 2 1 3 4	(D) 3 1 4 2

Q.68 The quadratic polynomial in x whose zeros are a, 2a is -
(A) (x + a) (x − 2a)
(B) (x − 2a) (x + 2a)
(C) (x + a) (x + 2a)
(D) (x − a) (x − 2a)

Q.69 The expression $x^4 + 7x^2 + 16$ can be factorized as-
(A) $(x^2 + x + 1)(x^2 + x + 16)$
(B) $(x^2 + x + 1)(x^2 - x + 16)$
(C) $(x^2 + x - 4)(x^2 - x + 4)$
(D) $(x^2 + x - 4)(x^2 - x - 4)$

Q.70 The solution of $2 - x = \frac{x-2}{x}$ would include-
(A) −2, −1 (B) 2, − 1
(C) −4, 2 (D) 4, −2

Q.71 The common roots of the equations $x^2 - 7x + 10 = 0$ and $x^2 - 10x + 16 = 0$ is -
(A) −2 (B) 3
(C) 5 (D) 2

Q.72 Let $f(x) = ax^2 + bx + c$. Then, match the following :
a. Sum of roots of $f(x) = 0$
1. $\frac{c}{a}$
b. Product of roots of $f(x) = 0$
2. $\frac{-b}{a}$
c. Roots of $f(x) = 0$ are real &
3. $b^2 - 4ac = 0$ distinct
d. Roots of $f(x) = 0$ are real and
4. $b^2 - 4ac < 0$ identical.
The correct matching is -

	a	b	c	d		a	b	c	d
(A)	2	1	4	3	(B)	1	2	3	4
(C)	4	3	1	2	(D)	3	4	1	2

Q.73 Let $f(x) = x^2 - 2qx - 1$. Match the following-
a. Sum of roots of $f(x) = 0$
1. 2q
b. Product of roots of $f(x) = 0$
2. −1
c. The roots of $f(x) = 0$ are both equal
3. q = − 1
4. Never

	a	b	c		a	b	c
(A)	2	1	4	(B)	2	1	3
(C)	1	2	4	(D)	1	2	3

Q.74 The sum of the roots of the equation $x^2 - 6x + 2 = 0$ is -
(A) −6 (B) −2
(C) 2 (D) 6

Q.75 If the product of the roots of $x^2 - 3x + k = 10$ is −2 the value of k is -
(A) −2 (B) 8
(C) 12 (D) −8

Q.76 If one root of the equation $2x^2 + ax + 6 = 0$ is 2, then a equals -
(A) 7 (B) $\frac{7}{2}$
(C) −7 (D) $\frac{-7}{2}$

Q.77 The ratio of the sum and the product of the roots of $7x^2 - 12x + 18 = 0$ is
(A) 7 : 12 (B) 2 : 3
(C) 3 : 2 (D) 7 : 18

Q.78 The roots of $2x^2 - 6x + 7 = 0$ are -
(A) real, unequal and rational
(B) real, unequal and irrational
(C) real and equal
(D) imaginary

Q.79 The roots of $2x^2 - 6x + 3 = 0$ are -
(A) real, unequal and rational
(B) real, unequal and irrational
(C) real and equal
(D) imaginary

Q.80 With respect to the roots of $x^2 - x - 2 = 0$, we can say that -
(A) both of them are natural numbers
(B) both of them are integers
(C) the latter of the two is negative
(D) None of these

Q.81 The equation $x^2 + 4x + k = 0$ has real roots, then-
(A) $k \geq 4$ (B) $k \leq 4$
(C) $k \leq 0$ (D) $k \geq 0$

Q.82 The value of k for which $x^2 - 4x + k = 0$ has coincident roots is -
(A) 4 (B) −4
(C) 0 (D) −2

Q.83 The equation (m being real), $mx^2 + 2x + m = 0$ has two distinct roots if-
(A) $m \neq 0$ (B) $m \neq 0, 1$
(C) $m \neq 1, -1$ (D) $m \neq 0, 1, -1$

Q.84 If the equation $x^2 + 2(k + 2)x + 9k = 0$ has equal roots, the values of k are -
(A) 1, 4 (B) −1, 4
(C) 1, −4 (D) −1, −4

Q.85 If the roots of $ax^2 + bx + c = 0$ be equal, then the value of c is -
(A) $\frac{-b}{2a}$ (B) $\frac{b}{2a}$
(C) $\frac{-b^2}{4a}$ (D) $\frac{b^2}{4a}$

Q.86 If the roots of $x^2 + 4mx + 4m^2 + m + 1 = 0$ are real, then -
(A) $m = -1$ (B) $m \le -1$
(C) $m \ge -1$ (D) $m \ge 0$

Q.87 The roots of the equation $(q - r)x^2 + (r - p)x + (p - q) = 0$ are-
(A) $\frac{r-p}{q-r}$, 1 (B) $\frac{p-q}{q-r}$, 1
(C) $\frac{q-r}{p-q}$, 1 (D) $\frac{r-p}{p-q}$, 1

Q.88 Vidhya and Vandana solved a quadratic equation. In solving it, Vidhya made a mistake in the constant term and got the roots as 6 and 2, while Vandana made a mistake in the coefficient of x only and obtained the roots as –7 and –1. The correct roots of the equation are -
(A) 6, – 1 (B) –7, 2
(C) –6, –2 (D) 7, 1

Q.89 A and B solved a quadratic equation. In solving it, A made a mistake in the constant term and obtained the roots as 5, – 3 while B made a mistake in the coefficient of x and obtained the roots as 1, –3. The correct roots of the equation are -
(A) 1, 3 (B) –1, 3
(C) –1, –3 (D) 1, –1

Q.90 If the equation $9x^2 + 6kx + 4 = 0$ has equal roots, then the value of k must be -
(A) zero
(B) either 2 or zero
(C) either –2 or zero
(D) either 2 or –2

Q.91 The roots of $\frac{x+4}{x-4} + \frac{x-4}{x+4} = \frac{10}{3}$ are -
(A) ± 4 (B) ± 6
(C) ± 8 (D) $2 \pm \sqrt{3}$

Q.92 The relationship between x and y as shown in the given table is :

x	0	1	2	3	4
y	100	90	70	40	0

(A) $y = 100 - 10x$
(B) $y = 100 - 5x - 5x^2$
(C) $y = 100 - 5x^2$
(D) $y = 20 - x - x^2$

Q.93 The roots of a quadratic equation are 5 and –2. The equation is -
(A) $x^2 - 3x + 10 = 0$
(B) $x^2 - 3x - 10 = 0$
(C) $x^2 + 3x + 10 = 0$
(D) $x^2 + 3x - 10 = 0$

Q.94 If the sum of the roots of a quadratic equation is 6 and the product of the roots is also 6, then the equation is -
(A) $x^2 - 6x + 6 = 0$
(B) $x^2 + 6x - 6 = 0$
(C) $x^2 - 6x - 6 = 0$
(D) $x^2 + 6x + 6 = 0$

Q.95 If one root of the equation $3x^2 - 10x + 3 = 0$ is $\frac{1}{3}$, the other root is-
(A) $-\frac{1}{3}$ (B) –3
(C) 3 (D) $\frac{1}{3}$

Q.96 If the equation $ax^2 - 5x + c = 0$ has 10 as the sum of the roots and also as the product of the roots, which of the following is true?
(A) $a = \frac{1}{2}$, $c = 5$ (B) $a = 2$, $c = 3$
(C) $a = 5$, $c = \frac{1}{2}$ (D) $a = 3$, $c = 2$

Q.97 If the sum of the roots of the equation $kx^2 + 2x + 3k = 0$ is equal to their product, then the value of k is -
(A) $\frac{1}{3}$ (B) $-\frac{1}{3}$
(C) $\frac{2}{3}$ (D) $-\frac{2}{3}$

Q.98 If α, β be the roots of $ax^2 + bx + c = 0$, the value of $\alpha^2 + \beta^2$ is -
(A) $\frac{b^2-4ac}{2a}$ (B) $\frac{b^2-2ac}{2a}$
(C) $\frac{b^2-2ac}{a^2}$ (D) $\frac{b^2-4ac}{2ac}$

Q.99 If α, β are the the roots $x^2 - px + q = 0$, then the value of $\alpha^2 + \beta^2$ is -
(A) $p^2 + 2q$ (B) $p^2 - 2q$
(C) $p(p^2 - 3q)$ (D) $p^2 - 4q$

Q.100 If α, β are the roots of the quadratic equation $x^2 - 6x + 6 = 0$, the value of $\alpha^2 + \beta^2$ is -
(A) 36 (B) 24
(C) 12 (D) 6

Q.101 If α, β are the roots of the equation $x^2 - 8x + p = 0$ and $\alpha^2 + \beta^2 = 40$, p is equal to -
(A) 8 (B) 10
(C) 12 (D) 14

Q.102 If α, β are the roots of the equation $x^2 + x + 1 = 0$, the value of $\alpha^4 + \beta^4$ is -
(A) 0 (B) 1
(C) –1 (D) None of these

Q.103 If α, β are the roots of the equations $2x^2 - 4x + 3 = 0$, then the value of $\alpha^3 + \beta^3$ is -
(A) –1 (B) 1
(C) 2 (D) 0

Q.104 If α, β are the roots of the equation $x^2 - 5x + 6 = 0$, the value of $\alpha^2 - \beta^2$ is -
(A) ± 4 (B) ± 5
(C) ± 6 (D) 0

Q.105 If α, β are the roots of $x^2 + px + q = 0$, the value of $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ is -
(A) $\frac{p^2-2q}{q}$ (B) $\frac{p^2+2q}{q}$
(C) $\frac{-p^2+2q}{q}$ (D) $\frac{-p^2-2q}{q}$

Q.106 If α, β are the roots of the equation $ax^2 + bx + c = 0$, the value of $\frac{\alpha}{\beta^2} + \frac{\beta}{\alpha^2}$ is-
(A) $\frac{b^2-2ac}{ac}$ (B) $\frac{-b^3+3abc}{a^3}$
(C) $\frac{-b^3+3abc}{ac^2}$ (D) $\frac{b^2-2ac}{a^2}$

Q.107 If one root of $5x^2 + 13x + k = 0$ be the reciprocal of the other root, the value of k is -
(A) 0 (B) 1
(C) 2 (D) 5

Q.108 The roots of the equation $ax^2 + bx + c = 0$ will be reciprocals if
(A) a = b (B) b = c
(C) c = a (D) None of these

Q.109 The value of k for which the roots α, β of the equation : $x^2 - 6x + k = 0$ satisfy the relation $3\alpha + 2\beta = 20$, is -
(A) 8 (B) –8
(C) 16 (D) –16

Q.110 If α, β are the roots of the equation $2x^2 - 3x + 1 = 0$, then the equation whose roots are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ is -
(A) $2x^2 + 5x + 2 = 0$
(B) $2x^2 - 5x - 2 = 0$
(C) $2x^2 + 5x - 2 = 0$
(D) $2x^2 - 5x + 2 = 0$

Q.111 If α, β are the roots of the equation $x^2 - 3x + 2 = 0$, then the equation whose roots are $(\alpha + 1)$ and $(\beta + 1)$ is
(A) $x^2 + 5x + 6 = 0$
(B) $x^2 - 5x - 6 = 0$
(C) $x^2 + 5x - 6 = 0$
(D) $x^2 - 5x + 6 = 0$

Q.112 If α, β are the roots of the equation $2x^2 - 5x + 7 = 0$, then the equation whose roots are $(2\alpha + 3\beta)$ and $(3\alpha + 2\beta)$ is -
(A) $2x^2 + 25x + 82 = 0$
(B) $2x^2 - 25x - 82 = 0$
(C) $2x^2 - 25x + 82 = 0$
(D) $2x^2 + 25x - 82 = 0$

Q.113 The quadratic equation whose roots are the reciprocals of the roots of the equation $x^2 - 20x + 17 = 0$, is -
(A) $20x^2 - 17x + 3 = 0$
(B) $17x^2 - 20x + 3 = 0$
(C) $20x^2 + 17x - 3 = 0$
(D) $17x^2 + 20x - 3 = 0$

Q.114 If α, β are the roots of the equation $x^2 + kx + 12 = 0$ such that $\alpha - \beta = 1$, the value of k is -
(A) 0 (B) ± 5
(C) ± 1 (D) ± 7

Q.115 The roots of the equation $x^2 + px + q = 0$ are 1 and 2. The roots of the equation $qx^2 - px + 1 = 0$ must be -
(A) 1, $\frac{1}{2}$ (B) $\frac{-1}{2}$, – 1
(C) $\frac{-1}{2}$, 1 (D) –1, $\frac{1}{2}$

Q.116 If the equations $x^2 + 2x - 3 = 0$ and $x^2 + 3x - k = 0$ have a common root, then the non-zero value of k is -
(A) 1 (B) 2
(C) 3 (D) 4

Q.117 If the equations $2x^2 - 7x + 3 = 0$ and $4x^2 + ax - 3 = 0$ have common root, then the values of a are -
(A) –11 or 4 (B) –11 or –4
(C) 11 or –4 (D) 11 or 4

Q.118 The positive value of m for which the roots of the equation $12x^2 + mx + 5 = 0$ are in the ratio 3 : 2 is -
(A) $5\sqrt{10}$ (B) $\frac{5}{2}\sqrt{10}$
(C) $\frac{5}{12}$ (D) $\frac{12}{5}$

Q.119 The solution set of the equation $x^{2/3} + x^{1/3} - 2 = 0$ is -
(A) {8, 1} (B) {8, –1}
(C) {–8, –1} (D) {–8, 1}

Q.120 Consider the following statements :
I. If the roots of the equation $ax^2 + bx + c = 0$ are negative reciprocal of each other, then $a + c = 0$.
II. A quadratic equation can have maximum two roots.
III. If α, β are the roots of a quadratic equation such that $\alpha + \beta = 22$ and $\alpha - \beta = 8$, then the equation $x^2 - 22x + 112 = 0$, has α and β as its roots.
IV. If α, β are the roots of the equation $2x^2 - 4x + 1 = 0$, the value of $\frac{1}{\alpha+2\beta} + \frac{1}{\beta+2\alpha}$ is $\frac{12}{17}$.
Of these statements.
(A) I, II and IV are correct
(B) I, III & IV are correct
(C) none is correct
(D) all are correct

Q.121 Match List I with List II. List I contains quadratic polynomials and List II contains the conditions for these polynomials to be fractorizable into a product of real linear factors.

List I	**List II**
a. $4x^2 + kx + 1$	1. $k \le \frac{1}{2}$
b. $kx^2 - 4x + k$	2. $k \ge 4$ or $k \le -4$
c. $kx^2 - 2x + 2$	3. $k \ge 8$ or $k \le 0$
d. $2x^2 - kx + k$	4. $-2 \le k \le 2$

a b c d a b c d
(A) 3 2 1 4 (B) 2 4 1 3
(C) 4 1 3 2 (D) 1 3 4 2

Q.122 The value of x in the equation $\sqrt{\frac{x}{1-x}} + \sqrt{\frac{1-x}{x}} = 2\frac{1}{6}$ is -
(A) $\frac{5}{13}$ (B) $\frac{7}{13}$
(C) $\frac{9}{13}$ (D) None of these

Q.123 The value of x in the equation $8\left(x^2+\frac{1}{x^2}\right) - 42\left(x-\frac{1}{x}\right) + 29 = 0$ is -
(A) 4 (B) –2
(C) $\frac{1}{2}$ (D) $\frac{1}{4}$

Q.124 The value of x in the equation $\sqrt{4x-3} + \sqrt{2x+3} = 6$ is -
(A) 3 (B) 1
(C) 100 (D) 111

Q.125 The two parts into which 57 should be divided so that their product is 782, are
(A) 43, 14 (B) 33, 24
(C) 34, 23 (D) 44, 13

Q.126 The sum of a number and its reciprocal is $2\frac{1}{20}$. the number is -
(A) $\frac{5}{4}$ (B) $\frac{3}{4}$
(C) $\frac{4}{3}$ (D) $\frac{1}{6}$

Q.127 A two digit number is such that the product of the digits is 8. When 18 is added to the number, the digits are reversed. The number is-
(A) 18 (B) 24
(C) 81 (D) 42

Q.128 The perimeter of a rectangle is 82 m and its area is 400 m^2. The breadth of the rectangle is -
(A) 25 m (B) 16 m
(C) 9 m (D) 20 m

Q.129 Out of a group of swans, $\left(\frac{7}{2}\right)$ times the square root of the number are swimming in water while two remaining are playing on the shore. The total numebr of swans is -
(A) 4 (B) 8
(C) 12 (D) 16

Q.130 If α and β are the roots of the equation $x^2 + x + 1 = 0$, the equation whose roots are α^{19} and β^7 is **[NTSE]**
(A) $x^2 - x - 1 = 0$ (B) $x^2 - x + 1$
(C) $x^2 + x - 1 = 0$ (D) $x^2 + x + 1 = 0$

Q.131 α, β are the roots of $ax^2 + bx + c = 0$, then the value of $\left\{\frac{1}{a\alpha+b} + \frac{1}{a\beta+b}\right\}$ **[NTSE]**
(A) $\frac{a}{bc}$ (B) $\frac{b}{ac}$
(C) $\frac{c}{ab}$ (D) None

Q.132 If α, β are the roots of $ax^2 - 2bx + c = 0$, then $\alpha^3\beta^3 + \alpha^2\beta^2 + \alpha^3\beta^2$ is **[NTSE]**
(A) $\frac{c^2(c+2b)}{a^3}$ (B) $\frac{bc^2}{a^3}$
(C) $\frac{c^2}{a^3}$ (D) None

Q.133 Ramesh and Mahesh solve an equation. In solving Ramesh commits a mistake in constant term and find the roots are 8 and 2. Mahesh commits a mistake in the coefficient of x and find the roots –9 and –1. The correct roots are **[NTSE]**
(A) –8, 2 (B) 8, 1
(C) 9, –1 (D) –8, –2

Q.134 If 8, 2 are the roots $x^2 + ax + b = 0$ and 3, 3 are the roots of $x^2 + ax + b = 0$, then the roots of the equation $x^2 + ax + b = 0$ are
(A) 8, –1 (B) –9, 2 **[NTSE]**
(C) –8, –2 (D) 9, 1

Q.135 Two students were solving a quadratic equation in x, one copied the constant term incorrectly and got the roots 3 and 2. The other copied the constant term and the coefficient of x^2 correctly as –6 and 1 respectively. The correct roots are **[NTSE]**
(A) 3, –2 (B) –3, 2
(C) –6, –6 (D) 6, –1

Q.136 If $\alpha + \beta = 3$, $\alpha^3 + \beta^3 = 7$, then α and β are the roots of **[NTSE]**
(A) $3x^2 + 9x + 7 = 0$
(B) $9x^2 - 27x + 20 = 0$
(C) $2x^2 - 6x + 15 = 0$
(D) None of these

Q.137 If α, β are the roots of $ax^2 + 2bx + c = 0$ and that of $Ax^2 + 2Bx + C = 0$ be $\alpha + \delta$, $\beta + \delta$, then the value of $\frac{b^2 - ac}{B^2 - AC}$ is **[NTSE]**
(A) $\left(\frac{a}{A}\right)^2$ (B) $\left(\frac{A}{a}\right)^2$
(C) 0 (D) 1

Q.138 The condition that the roots of the equation $ax^2 + bx + c = 0$ be such that one root is n times the other is **[NTSE]**
(A) $na^2 = bc(n + 1)^2$
(B) $nb^2 = ca(n + 1)^2$
(C) $nc^2 = ab(n + 1)^2$
(D) None of these

Q.139 If the root of the equation $ax^2 + bx + c = 0$ are in the ratio m : n, then **[NTSE]**
(A) $mna^2 = (m + n)c^2$
(B) $mnb^2 = (m + n)ac$
(C) $mnb^2 = (m + n)2ac$
(D) None of these

Q.140 The number of real roots of the equation $(x - 1)^2 + (x - 2)^2 + (x - 3)^2$ is **[NTSE]**
(A) 2 (B) 1
(C) 0 (D) 3

ANSWER KEY

1.	C	**2.**	D	**3.**	B	**4.**	C
5.	A	**6.**	B	**7.**	C	**8.**	D
9.	B	**10.**	D	**11.**	A	**12.**	B
13.	A	**14.**	D	**15.**	A	**16.**	D
17.	B	**18.**	C	**19.**	D	**20.**	B
21.	C	**22.**	B	**23.**	C	**24.**	C
25.	B	**26.**	C	**27.**	B	**28.**	D
29.	B	**30.**	B	**31.**	C	**32.**	B
33.	C	**34.**	B	**35.**	A	**36.**	D
37.	B	**38.**	D	**39.**	D	**40.**	C
41.	D	**42.**	C	**43.**	A	**44.**	D
45.	C	**46.**	C	**47.**	A	**48.**	B
49.	C	**50.**	C	**51.**	D	**52.**	B
53.	D	**54.**	D	**55.**	C	**56.**	A
57.	C	**58.**	C	**59.**	B	**60.**	C
61.	B	**62.**	B	**63.**	C	**64.**	C
65.	D	**66.**	D	**67.**	A	**68.**	D
69.	C	**70.**	B	**71.**	D	**72.**	A
73.	C	**74.**	D	**75.**	B	**76.**	C
77.	B	**78.**	D	**79.**	B	**80.**	B
81.	B	**82.**	A	**83.**	C	**84.**	A
85.	D	**86.**	B	**87.**	B	**88.**	D
89.	B	**90.**	D	**91.**	C	**92.**	B
93.	B	**94.**	A	**95.**	C	**96.**	A
97.	D	**98.**	C	**99.**	B	**100.**	B
101.	C	**102.**	C	**103.**	A	**104.**	B
105.	A	**106.**	C	**107.**	D	**108.**	C
109.	D	**110.**	D	**111.**	D	**112.**	C
113.	B	**114.**	D	**115.**	B	**116.**	D
117.	A	**118.**	A	**119.**	D	**120.**	A
121.	B	**122.**	C	**123.**	A	**124.**	D
125.	C	**126.**	A	**127.**	B	**128.**	B
129.	D	**130.**	D	**131.**	B	**132.**	A
133.	B	**134.**	D	**135.**	D	**136.**	B
137.	A	**138.**	B	**139.**	C	**140.**	C

ARITHMETIC PROGRESSIONS

INTRODUCTION

Consider the following arrangement of numbers :

(i) 1, 3, 5, 7, ... (ii) 3, 6, 12, 24, ... (iii) 1, 4, 9, 16, ...

In each of the above arrangements, we observe some patterns. In (i) we find that the succeeding terms are obtained by adding a fixed number [i.e. 2], in (ii) by multiplying with a fixed number [i.e. 2], in (iii) we find that they are squares of natural numbers.

In this chapter, we shall discuss one of these patterns in which succeeding terms are obtained by adding a fixed number to the preceding terms. We shall also see how to find their n^{th} terms and the sum of n consecutive terms, and use this knowledge in solving some daily life problems.

HISTORICAL FACTS

Gauss was a very talented and gifted mathematician of 19th century who developed the formula :

$1 + 2 + 3 + 4 + ... + (n - 1) + n = \frac{n(n+1)}{2}$ for the sum of first n natural numbers at the age of 10. He did this in the following way :

$S = 1 + 2 + 3 + ... + (n - 2) + (n - 1) + n$

$S = \underline{n + (n - 1) + (n - 2) + ...+ 3 + 2 + 1}$

$\therefore \quad 2S = (n + 1) + (n + 1) + (n + 1) + ...+ (n + 1) + (n + 1) + (n + 1)$

$= (n + 1) (1 + 1 + 1 + ...$ upto n times)

$2S = (n + 1)\, n \Rightarrow S = \frac{n(n+1)}{2}$

Even when he was a little child of three he could read and make mathematical calculation himself. Gauss proved the fundamental theorem of Algebra when he was 20 years old. His contribution to mathematics has been immense because his formulae were used in applied field of Astronomy, Differential Geometry and Electricity widely all over the world by scientists.

DEFINITION

When the terms of a sequence or series are arranged under a definite rule then they are said to be in a Progression. Progression can be classified into 5 parts as -

(i) Arithmetic Progression (A.P.)
(ii) Geometric Progression (G.P.)
(iii) Arithmetic Geometric Progression (A.G.P.)
(iv) Harmonic Progression (H.P.)
(v) Miscellaneous Progression

Arithmetic Progression (A.P.)

Arithmetic Progression is defined as a series in which difference between any two consecutive terms is constant throughout the series. This constant difference is called common difference. If 'a' is the first term and 'd' is the common difference, then an AP can be written as

$a + (a+ d) + (a + 2d) + (a + 3d) + ...$

Note: If a,b,c, are in AP $\Leftrightarrow$ $2b = a + c$

General Term of an AP

General term (n^{th} term) of an AP is given by

$T_n = a + (n- 1)\, d$

NOTE :

(i) General term is also denoted by ℓ (last term)
(ii) n (No. of terms) always belongs to set of natural numbers.
(iii) Common difference can be zero, +ve or –ve.
(iv) n^{th} term from end is given by

$= T_m - (n- 1)\, d$

or $= (m - n + 1)^{th}$ term from beginning where m is total no. of terms.

Sum of n terms of an A.P.

The sum of first n terms of an A.P. is given by

$S_n = \frac{n}{2}[2a + (n-1)\,d]$ or $S_n = \frac{n}{2}[a + T_n]$

NOTE :

(i) If sum of n terms S_n is given then general term $T_n = S_n - S_{n-1}$ where S_{n-1} is sum of (n–1) terms of A.P.

(ii) Common difference of AP is given by

$d = S_2 - 2S_1$ where S_2 is sum of first two terms and S_1 is sum of first term.

Arithmetic Mean (A.M.)

If three or more than three terms are in A.P., then the numbers lying between first and last term are known as Arithmetic Means between them.i.e.

The A.M. between the two given quantities a and b is A so that a, A, b are in A.P.

i.e. $A - a = b - A \Rightarrow A = \frac{a+b}{2}$

Note : A.M. of any n positive numbers a_1, a_2 .....a_n is

$$A = \frac{a_1 + a_2 + a_3 + \ldots . a_n}{n}$$

n AM's between two given numbers

If in between two numbers 'a' and 'b' we have to insert n AM A_1, A_2, ...A_n then a, A_1, A_2, A_3....A_n, b will be in A.P. The series consist of (n+2) terms and the last term is b and first term is a.

$\Rightarrow$ $a + (n + 2 - 1)\,d = b$

$\Rightarrow$ $d = \frac{b-a}{n+1}$

$A_1 = a + d$, $A_2 = a + 2d$,.... $A_n = a + nd$ or $A_n = b - d$

Note :

(i) Sum of n AM's inserted between a and b is equal to n times the single AM between a and b

i.e. $\sum_{r=1}^{n} A_r = nA$ where $A = \frac{a+b}{2}$

(ii) between two numbers

$\frac{\text{sum of m AM's}}{\text{sum of n AM's}} = \frac{m}{n}$

Supposition of Terms in A.P.

(i) When no. of terms be odd then we take three terms are as: a – d, a, a+ d five terms are a – 2d, a – d, a, a+ d, a + 2d

Here we take middle term as 'a' and common difference as 'd'.

(ii) When no. of terms be even then we take 4 term are as :

a – 3d, a– d, a + d, a + 3d

6 term are as = a – 5d, a – 3d, a – d, a+ d, a + 3d, a + 5d

Here we take 'a – d, a + d' as middle terms and common difference as '2d'.

NOTE :

(i) If no. of terms in any series is odd then only one middle term is exist which is $\left(\frac{n+1}{2}\right)^{th}$ term where n is odd.If no. of terms in any series is even then middle terms are two which are given

(ii) by $(n/2)^{th}$ and $\left(\frac{n}{2}+1\right)^{th}$ term where n is even.

Some Properties of an A.P.

(i) If each term of a given A.P. be increased, decreased, multiplied or divided by some non zero constant number then resulting series thus obtained will also be in A.P.

(ii) In an A.P., the sum of terms equidistant from the beginning and end is constant and equal to the sum of first and last term.

(iii) Any term of an AP (except the first term) is equal to the half of the sum of terms equidistant from the term i.e.

$$a_n = \frac{1}{2}\left(a_{n-k} + a_{n+k}\right),\ k < n$$

(iv) If in a finite AP, the number of terms be odd, then its middle term is the AM between the first and last term and its sum is equal to the product of middle term and no. of terms.

Some Standard Results

(i) Sum of first n natural numbers

$$\Rightarrow \quad \sum_{r=1}^{n} r = \frac{n(n+1)}{2}$$

(ii) Sum of first n odd natural numbers

$$\Rightarrow \quad \sum_{r=1}^{n} (2r-1) = n^2$$

(iii) Sum of first n even natural numbers

$$= \sum_{r=1}^{n} 2r = n(n+1)$$

(iv) Sum of squares of first n natural numbers

$$= \sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}$$

(v) Sum of cubes of first n natural numbers

$$= \sum_{r=1}^{n} r^3 = \frac{n^2(n+1)^2}{4}$$

(vi) If r^{th} term of an A.P.

$T_r = Ar^3 + Br^2 + Cr + D$, then sum of n term of AP is

$$S_n = \sum_{r=1}^{n} T_r = A\sum_{r=1}^{n} r^3 + B\sum_{r=1}^{n} r^2 + C\sum_{r=1}^{n} r + D\sum_{r=1}^{n} 1$$

SUMMARY OF THE CHAPTER

BASIC CONCEPTS AND IMPORTANT RESUTLS

*** Sequence**

A collection of numbers arranged in a definite order according to some definite rule (rules) is called a sequence. Each number of the sequence is called a term of the sequence. The sequence is called finite or infinite according as the number of terms in it is finite or infinite.

The different terms of a sequence are usually denoted by a_1, a_2, a_3, ... or by t_1, t_2, t_3, ... The subscript (always a natural number) denotes the position of the term in the sequence. The number occuring at the nth place of a sequence i.e. a_n is called the general term of the sequence.

For example, consider the following lists of numbers :

(i) 5, 7, 9, 11, ... (ii) 11, 8, 5, 2, –1,

(iii) 2, 6, 18, 54, ... (iv) 1, 4, 9, 16,

We observe as under :

(i) Here each term is obtained by adding 2 to the preceding term.

(ii) Here each term is obtained by subtracting 3 from the preceding term i.e. by adding –3 to the preceding term.

(iii) Here each term is obtained by multiplying the preceding term by 3.

(iv) Here each term is obtained by squaring the next natural number.

Each one of the above list of numbers is a sequence.

*** SERIES**

If the terms of a sequence are connected by plus signs we get a series.

Thus, if a_1, a_2, a_3, ... is a given sequence, then the expression $a_1 + a_2 + a_3 + ...$ is called the series associated with the given sequence.

For sequences (i) to (iv) given above, we get the following series :

(i) $5 + 7 + 9 + 11 + ...$ (ii) $11 + 8 + 5 + 2 + (-1) + ...$

(iii) $2 + 6 + 18 + 54 + ...$ (iv) $1 + 4 + 9 + 16 + ...$

*** Arithmetic progression**

A sequence is called an arithmetic progression (abbreviated A.P.) if and only if the difference of any term from its preceding term is constant.

This constant is usually denoted by d and is called common difference. Thus a_1, a_2, a_3, a_4, ... is an A.P. if and only if $a_{n+1} - a_n = d$, a constant (independent of n). It follows that, in an A.P., $a_{n+1} = a_n + d$ i.e. any term (except the first) is obtained by adding the fixed number d to its preceding term.

The sequences (i) and (ii) mentioned earlier are arithmetic progressions.

In an A.P., note that :

(i) first term is any arbitrarily chosen real number.

(ii) every successive term can be obtained by adding the same fixed number to its preceding term (fixed number is any real number, positive, zero or negative).

To show that a given sequence is an A.P., we have to prove that $a_{n+1} - a_n$ is constant (independent of n).

*** General term of an A.P.**

Let a be the first term and d be the common difference of an A.P., then the A.P. is a, a + d, a + 2d, ... and its nth term = a + (n – 1)d.

Hence general term $a_n = a + (n - 1)d$.

* **Last term of an A.P.**

If the last term of an A.P. a, a + d, a + 2d, ..., consisting of n terms is denoted by ℓ, then ℓ = a + (n – 1) d.

* **The nth term from the end of an A.P.**

(i) If a, a + d, a + 2d, ... is an A.P. consisting of m terms, then the nth term from the end

= $\left(m - \overline{n-1}\right)$th term from the beginning

= (m – n + 1)th term from beginning

= a + $\left(\overline{m-n+1}-1\right)$d = a + (m – n)d.

(ii) If a, a + d, a + 2d, ... is an A.P. with last term ℓ, then the nth term from the end = ℓ + (n – 1) (–d) = ℓ – (n – 1)d

Because when we look at the terms of the given A.P. from last and move towards beginning we find that the sequence is an A.P. with common difference –d and first term as ℓ, therefore, the nth term from the end of the given A.P.

= ℓ + (n – 1)(–d) = ℓ – (n – 1)d

Thus, nth term from end = ℓ – (n – 1)d

* **Difference of any two terms of an A.P.**

Let a, a + d, a + 2d, ... be the given A.P., then n^{th} term – mth term = [a + (n – 1)d]

– [a + (m – 1)d] = (n – m)d

Thus, $a_n - a_m = (n - m)d$

* **Sum of n terms of an A.P.**

Let a be the fist term, d the common difference and ℓ the last term of an A.P. If S_n denotes the sum of first n terms, then

$S_n = \frac{n}{2}[2a + (n-1)d]$ or $S_n = \frac{n}{2}(a + \ell)$

Also $S_n - S_{n-1}$ = sum of first n terms – sum of first (n – 1) terms

= nth term = a_n.

Thus, $a_n = S_n - S_{n-1}$.

* **Number in A.P.**

1. Three numbers a, b and c are in A.P. if and only if 2b = a + c.

2. Some problems involve 3, 4 or 5 numbers in A.P.

If the sum of the numbers is given, then in an A.P.,

(i) three numbers are taken as a – d, a, a + d.

(ii) four numbers are taken as a – 3d, a – d, a + d, a + 3d.

(iii) five numbers are taken as a – 2d, a – d, a, a + d, a + 2d etc.

SOLVED PROBLEMS

Ex.1 Write the first five terms of the sequence, whose nth term is $a_n = \{1 + (-1)^n\}n$.

Sol. $a_n = \{1 + (-1)^n\}n$

Substituting n = 1, 2, 3, 4 and 5, we get

$a_1 = \{1 + (-1)^1\}\ 1 = 0$; $a_2 = \{1 + (-1)^2\}\ 2 = 4$;

$a_3 = \{1 + (-1)^3\}\ 3 = 0$; $a_4 = \{1 + (-1)^4\}\ 4 = 8$;

$a_5 = \{1 + (-1)^5\}\ 5 = 0$

Thus, the required terms are : 0, 4, 0, 8 and 0.

Ex.2 Find the 20th term of the sequence whose nth term is, $a_n = \frac{n(n-2)}{n+3}$

Sol. $a_n = \frac{n(n-2)}{n+3}$. Putting n = 20, we obtain $a_{20} = \frac{20(20-2)}{20+3}$

Thus, $a_{20} = \frac{360}{23}$

Ex.3 The fibonacci sequence is defined by $a_1 = 1 = a_2$; $a_n = a_{n-1} + a_{n-2}$ for n > 2. Find $\frac{a_{n+1}}{a_n}$, for n = 1, 2, 3, 4, 5.

Sol. We have $a_1 = a_2 = 1$ and $a_n = a_{n-1} + a_{n-2}$

Substituting n = 3, 4, 5 and 6, we get.

$a_3 = a_2 + a_1 = 1 + 1 = 2$

$a_4 = a_3 + a_2 = 2 + 1 = 3$

$a_5 = a_4 + a_3 = 3 + 2 = 5$

and $a_6 = a_5 + a_4 = 5 + 3 = 8$

Now, we have to find $\frac{a_{n+1}}{a_n}$ for n = 1, 2, 3, 4 and 5

For, n = 1, $\frac{a_2}{a_1} = \frac{1}{1} = 1$, n = 2, $\frac{a_3}{a_2} = \frac{2}{1} = 2$,

n = 3, $\frac{a_4}{a_3} = \frac{3}{2}$, n = 4, $\frac{a_5}{a_4} = \frac{5}{3}$,

n = 5, $\frac{a_6}{a_5} = \frac{8}{5}$

Hence, the required values are 1, 2, $\frac{3}{2}, \frac{5}{3}$ and $\frac{8}{5}$

Ex.4 In which of the following situations, does the list of numbers involved make an arithmetic progression, and why? **[NCERT]**

(i) The taxi fare after each km when the fare is Rs. 15 for the first km and Rs 8 for each additional km.

(ii) The amount of air present in a cylinder when a vacuum pump removes $\frac{1}{4}$ of the air remaining in the cylinder at a time.

(iii) The cost of digging a well after every metre of digging, when it costs Rs. 150 for the first metre and rises by Rs. 50 for each subsequent metre.

(iv) The amount of money in the account every year, when Rs. 10000 is deposited at compound interest at 8% per annum.

Sol. (i) Taxi fare for 1 km = Rs.15 = a_1
Taxi fare for 2 kms = Rs.15+8 = Rs.23=a_2
Taxi fare for 3 kms = Rs.23+8 = Rs.31=a_3
Taxi fare for 4 kms = Rs.31+8 = Rs.39=a_4 and so on.
$a_2 - a_1$ = Rs. 23 – 15 = Rs. 8
$a_3 - a_2$ = Rs. 31 – 23 = Rs. 8
$a_4 - a_3$ = Rs. 39 – 31 = Rs. 8
i.e., $a_{k+1} - a_k$ is the same everytime.
So, this list of numbers form an arithmetic progression with the first term a = Rs 15 and the common difference d = Rs. 8.

(ii) Amount of air present in the cylinder = x units (say) = a_1

Amount of air present in the cylinder after one time removal of air by the vacuum pump = $x - \frac{x}{4} = \frac{3x}{4}$ units = a_2

Amount of air present in the cylinder after two times removal of air by the vacuum pump = $\frac{3x}{4} - \frac{1}{4}\left(\frac{3x}{4}\right) = \frac{3x}{4} - \frac{3x}{16} = \frac{9x}{16}$ units = $\left(\frac{3}{4}\right)^2 x$ units = a_3

Amount of air present in the cylinder after three times removal of air by the vacuum pump

$= \left(\frac{3}{4}\right)^2 x - \frac{1}{4}\left(\frac{3}{4}\right)^2 x \quad \Rightarrow \left(1 - \frac{1}{4}\right)\left(\frac{3}{4}\right)^2 x$

$\Rightarrow \left(\frac{3}{4}\right)\left(\frac{3}{4}\right)^2 x = \left(\frac{3}{4}\right)^3 x$ units = a_4 and so on.

$a_2 - a_1 = \frac{3x}{4} - x = \frac{x}{4}$ units

$a_3 - a_2 = \left(\frac{3}{4}\right)^2 x - \frac{3}{4} x = -\frac{3}{16} x$ units

As $a_2 - a_1 \neq a_3 - a_2$, this list of numbers does not form an AP.

(iii) Cost of digging the well after 1 metre of digging = Rs. 150 = a_1
Cost of digging the well after 2 metres of digging = Rs. 150 + 50 = Rs 200 = a_2
Cost of digging the well after 3 metres of digging = Rs. 200 + 50 = Rs 250 = a_3
Cost of digging the well after 4 metres of digging = Rs. 250 + 50 = Rs 300 = a_4 and so on.
$a_2 - a_1$ = Rs 200 – 150 = 50
$a_3 - a_2$ = Rs 250 – 200 = 50
$a_4 - a_3$ = Rs 300 – 250 = 50
i.e., $a_{k-1} - a_k$ is the same everytime. So this list of numbers forms an AP with the first term a = Rs. 150 and the common difference d = Rs. 50

(iv) Amount of money after 1 year = Rs.

$$10000\left(1 + \frac{8}{100}\right) = a_1$$

Amount of money after 2 years = Rs.

$$10000\left(1 + \frac{8}{100}\right)^2 = a_2$$

Amount of money after 3 years = Rs.

$$10000\left(1 + \frac{8}{100}\right)^3 = a_3$$

Amount of money after 4 years = Rs.

$$10000\left(1 + \frac{8}{100}\right)^4 = a_4$$

$a_2 - a_1$ = Rs. $10000\left(1 + \frac{8}{100}\right)^2$ – Rs. $10000\left(1 + \frac{8}{100}\right)$

$= \text{Rs. } 10000\left(1+\frac{8}{100}\right)\left(1+\frac{8}{100}-1\right)$

$= \text{Rs. } 10000\left(1+\frac{8}{100}\right)\left(\frac{8}{100}\right)$

$a_3-a_2 = \text{Rs.}10000\left(1+\frac{8}{100}\right)^3 - \text{Rs.}10000\left(1+\frac{8}{100}\right)^2$

$\Rightarrow \text{Rs. } 10000\left(1+\frac{8}{100}\right)^2\left(1+\frac{8}{100}-1\right)$

$= \text{Rs. } 10000\left(1+\frac{8}{100}\right)^2\left(\frac{8}{100}\right)$

As $a_2-a_1 \neq a_3-a_2$, this list of numbers does not form an AP.

Ex.5 Write first four terms of the AP, when the first term a and the common difference d are given as follows: **[NCERT]**

(i) a = 4, d = 5 (ii) a = –1.25, d=– 0.25

Sol. (i) a = 4, d = 5

First term, a = 4

Second term = 4 + d = 4 + 5 = 9

Third term = 9 + d = 9 + 5 = 14

Fourth term = 14 + d = 14 + 5 = 19

Hence, first four terms of the given AP are 4, 9, 14, 19.

(ii) a = – 1.25, d = – 0.25

First term = a = – 1.25

Second term = –1.25+d =–1.25+(–0.25)= –1.50

Third term = –1.50+d = –1.50+(–0.25) = –1.75

Fourth term = –1.75+d = –1.75+(–0.25) = –2.00

Hence, first four terms of the given AP are –1.25, –1.50, –1.75, –2.00

Ex.6 For the AP $\frac{3}{2},\frac{1}{2},-\frac{1}{2},-\frac{3}{2},\ldots$ write the first term a and the common difference d. Also write the next two terms after the given last term $-\frac{3}{2}$. **[NCERT]**

Sol. We have, $a_1 = \frac{3}{2}$, $a_2 = \frac{1}{2}$, $a_3 = -\frac{1}{2}$, $a_4 = -\frac{3}{2}$ and so on. Thus, $a = \frac{3}{2}$

$a_2 - a_1 = \left(\frac{1}{2}\right) - \left(\frac{3}{2}\right) = -1,$

$a_3 - a_2 = \left(-\frac{1}{2}\right) - \left(\frac{1}{2}\right) = -1,$

$a_4 - a_3 = \left(-\frac{3}{2}\right) - \left(-\frac{1}{2}\right) = -1,$ and so on.

$\Rightarrow$ d = – 1

Now, we find the successor of $-\frac{3}{2}$.

$a_5 = \left(-\frac{3}{2}\right) + d = \left(-\frac{3}{2}\right) + (-1) = -\frac{5}{2}$

Then $a_6 = a_5 + d = \left(-\frac{5}{2}\right) + (-1) = -\frac{7}{2}$

Hence, the next two terms after the given term $-\frac{3}{2}$ are $-\frac{5}{2}, -\frac{7}{2}$.

Ex.7 Find the 30th term of the AP : 10, 7, 4.... **[NCERT]**

Sol. The given A.P. is 10, 7, 4,....

Here, a = 10, d = 7 – 10 = – 3 and n = 30

we have $a_n = a + (n - 1)d$

So, $a_{30} = 10 + (30 - 1)(-3)$

$\Rightarrow a_{30} = 10 - 87 \Rightarrow a_{30} = -77$

$\therefore$ The 30th term of the given AP is – 77.

Ex.8 The 6th term of an arithmetic progression is –10 and the 10th term is –26. Determine the 15th term of the AP.

Sol. Let the first term and the common difference of the AP be a and d respectively.

6th term = – 10 (Given)

$\Rightarrow a+(6-1)d = -10$ [$\because a_n = a+(n-1)d$]

$\Rightarrow a + 5d = -10$ ...(i)

10th term = – 26 (Given)

$\Rightarrow a + (10-1)d = -26$

$\Rightarrow a + 9d = -26$ ...(ii)

Solving (i) and (ii) we get a = 10, d = – 4

Therefore, 15th term of the AP

$= a + (15 - 1)d$ [$\because a_n = a + (n - 1)d$]

$= a + 14d = 10 + 14(-4) = 10 - 56 = -46$

Hence, the 15th term of AP is – 46.

Ex.9 Find the 6th term from the end of the AP 17, 14, 11,...., – 40.

Sol. The given AP 17, 14, 11,..., – 40

Here, a = 17, d = 14 – 17 = – 3, $\ell = -40$

Let there be n terms in the given AP.

Then, nth term = –40

$\Rightarrow a + (n - 1)d = -40$ [$\because a_n = a + (n - 1)d$]

$\Rightarrow 17+(n-1)(-3) = -40$

$\Rightarrow (n-1)(-3) = -40-17$

$\Rightarrow (n-1)(-3) = -57 \Rightarrow n = 20$

Hence, there are 20 terms in the given AP. Now, 6th term from the end

$= a+(20-6)d$ [$\because$ rth term from the end = a + (n – r)d]

$= a + 14d = 17 + 14(-3) = 17 - 42 = -25$

Hence, the 6th term from the end of the given AP is – 25.

Ex.10 Is 200 any term of the sequence 3, 7, 11, 15,...?

Sol. The given sequence is 3, 7, 11, 15,...

$a_2 - a_1 = 7 - 3 = 4$

$a_3 - a_2 = 11 - 7 = 4$

$a_4 - a_3 = 15 - 11 = 4$

As $a_{k+1} - a_k$ is the same for k = 1, 2, 3, etc., the given sequence form an AP.

Here, a = 3, d = 4

Let 200 be the nth term of the given sequence. Then,

$a_n = 200$

$\Rightarrow a + (n - 1)d = 200 \Rightarrow 3 + (n - 1)4 = 200$

$\Rightarrow (n - 1) = \frac{197}{4} \Rightarrow n = \frac{197}{4} + 1 \Rightarrow n = \frac{201}{4}$.

But n should be a positive integer. So, 200 is not any term of the given sequence.

Ex.11 The sum of three numbers in AP is –3, and their product is 8. Find the numbers.

Sol. Let the numbers be (a – d), a, (a + d). Then,

Sum = – 3 $\Rightarrow$ (a – d) + a + (a + d) = – 3 $\Rightarrow$ 3a = – 3 $\Rightarrow$ a = – 1

Now, product = 8

$\Rightarrow$ (a –d)(a)(a + d) = 8

$\Rightarrow$ $a(a^2 - d^2) = 8$

$\Rightarrow$ $(-1)(1 - d^2) = 8$ [$\because$ a = –1]

$\Rightarrow$ $d^2 = 9 \Rightarrow d = \pm 3$

If d = 3, the numbers are –4, –1, 2. If d = –3, the numbers are 2, – 1, – 4

Thus, the numbers are –4, –1, 2 or 2, –1, –4

Ex.12 Find four numbers in AP, whose sum is 20 and the sum of whose squares is 120.

Sol. Let the numbers be (a–3d), (a–d), (a+d), (a+3d). Then,

Sum = 20

$\Rightarrow$ (a–3d)+(a–d)+(a+d)+(a+3d)=20

$\Rightarrow$ 4a = 20 $\Rightarrow$ a = 5

Now sum of the squares = 120

$\Rightarrow$ $(a-3d)^2 + (a-d)^2 + (a+d)^2 + (a+3d)^2 = 120$

$\Rightarrow$ $4a^2+20d^2=120$ $\Rightarrow$ $a^2 + 5d^2 = 30$

$\Rightarrow$ $25 + 5d^2 = 30$ $\Rightarrow$ $25 + 5d^2 = 30$

$\Rightarrow$ $5d^2 = 5$ $\Rightarrow$ d = ± 1

If d=1, then the numbers are 2, 4, 6, 8. If d =–1, then the numbers are 8, 6, 4, 2.

Thus, the numbers are 2, 4, 6, 8 or 8, 6, 4, 2.

Ex.13 If 2x, x + 10, 3x + 2 are in AP. Find the value of x.

Sol. Since, 2x, x + 10, 3x + 2 are in AP.

$\therefore$ 2 (x + 10) = 2x + (3x + 2)

$\Rightarrow$ 2x + 20 = 5x + 2 $\Rightarrow$ 3x = 18

$\Rightarrow$ x = 6.

Ex.14 Find the sum of the AP: $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}$,...., to 11 terms. **[NCERT]**

Sol. Here, $a = \frac{1}{15}$

$d = \frac{1}{12} - \frac{1}{15} = \frac{1}{60}$

n = 11

We know that

$\Rightarrow S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow S_{11} = \frac{11}{2}\left[2\left(\frac{1}{15}\right) + (11-1)\left(\frac{1}{60}\right)\right]$

$\Rightarrow S_{11} = \frac{11}{2}\left[\frac{2}{15} + \frac{1}{6}\right]$

$\Rightarrow S_{11} = \frac{11}{2}\left[\frac{3}{10}\right]$ $\Rightarrow S_{11} = \frac{33}{20}$

So, the sum of the first 11 terms of the given AP is $\frac{33}{20}$.

Ex.15 Find the sum : 34 + 32 + 30 + ... + 10 **[NCERT]**

Sol. 34 + 32 + 30 + ... + 10

This is an AP

Here, $a = 34$

$d = 32 - 34 = -2$

$\ell = 10$

Let the number of terms of the AP be n.

We know that

$a_n = a + (n-1)d$

$\Rightarrow 10 = 34+(n-1)(-2) \Rightarrow (n-1)(-2) = -24$

$\Rightarrow n - 1 = \frac{-24}{-2} = 12 \Rightarrow n = 13$

Again, we know that

$S_n = \frac{n}{2}(a+\ell) \Rightarrow S_{13} = \frac{13}{2}(34+10) \Rightarrow S_{13} = 286$

Hence, the required sum is 286.

Ex.16 Find the sum of all natural numbers between 100 and 200 which are divisible by 4.

Sol. All natural numbers between 100 and 200 which are divisible by 4 are

104, 108, 112, 116,...,196

Here, $a_1 = 104$

$a_2 = 108$

$a_3 = 112$

$a_4 = 116$

$\therefore a_2 - a_1 = 108 - 104 = 4$

$a_3 - a_2 = 112 - 108 = 4$

$a_4 - a_3 = 116 - 112 = 4$

$\because a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = ... (= 4 \text{ each})$

$\therefore$ This sequence is an arithmetic progression whose common difference is 4.

Here, $a = 104$

$d = 4 \quad \ell = 196$

Let the number of terms be n. Then

$\ell = a + (n-1)d$

$\Rightarrow 196 = 104 + (n-1)4$

$\Rightarrow 196 - 104 = (n-1)4$

$\Rightarrow 92 = (n-1)\,4 \Rightarrow (n-1)\,4 = 92$

$\Rightarrow n - 1 = \frac{92}{4} \Rightarrow n - 1 = 23$

$\Rightarrow n = 23 + 1 \Rightarrow n = 24$

Again, we know that

$S_n = \frac{n}{2}(a + \ell)$

$\Rightarrow S_{24} = \left(\frac{24}{2}\right)(104 + 196) = (12)(300) = 3600$

Hence, the required sum is 3600.

Ex.17 Find the number of terms of the AP 54, 51, 48,...so that their sum is 513.

Sol. The given AP is 54, 51, 48,...

Here, a = 54, d = 51– 54 = – 3

Let the sum of n terms of this AP be 513.

We know that

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow 513 = \frac{n}{2}[2(54) + (n-1)(-3)]$

$\Rightarrow 513 = \frac{n}{2}[108 - 3n + 3]$

$\Rightarrow 513 = \frac{n}{2}[111 - 3n]$

$\Rightarrow 1026 = n[111 - 3n]$

$\Rightarrow 1026 = 111n - 3n^2$

$\Rightarrow 3n^2 - 111n + 1026 = 0$

$\Rightarrow n^2 - 37n + 342 = 0$ [Dividing throughout by 3]

$\Rightarrow n^2 - 18n - 19n + 342 = 0$

$\Rightarrow n(n-18) - 19(n-18) = 0$

$\Rightarrow (n-18)(n-19) = 0$

$\Rightarrow n - 18 = 0$ or $n - 19 = 0$

$\Rightarrow n = 18, 19$

Hence, the sum of 18 terms or 19 terms of the given AP is 513.

Note : Actually 19th term

$= a_{19}$

$= a + (19-1)d \qquad [\because a_n = a + (n-1)d]$

$= a + 18d$

$= 54 + 18(-3)$

$= 54 - 54 = 0$

Ex.18 Find the AP whose sum to n terms is $2n^2 + n$.

Sol. Here, $S_n = 2n^2 + n$(Given)

Put n = 1, 2, 3, 4,..., in succession, we get

$S_1 = 2(1)^2 + 1 = 2 + 1 = 3$

$S_2 = 2(2)^2 + 2 = 8 + 2 = 10$

$S_3 = 2(3)^2 + 3 = 18 + 3 = 21$

$S_4 = 2(4)^2 + 4 = 32 + 4 = 36$

and so on.

$\therefore a_1 = S_1 = 3$

$a_2 = S_2 - S_1 = 10 - 3 = 7$

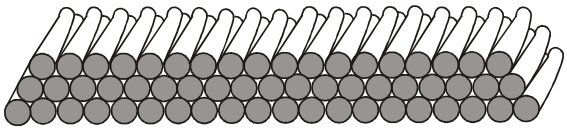
$a_3 = S_3 - S_2 = 21 - 10 = 11$

$a_4 = S_4 - S_3 = 36 - 21 = 15$

and so on.

Hence, the required AP is 3, 7, 11, 15,...

Ex.19 200 logs are stacked in the following manner : 20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on (see figure). In how many rows are the 200 logs placed and how many logs are in the top row? **[NCERT]**



Sol. The number of logs in the bottom row, next row, row next to it and so on form the sequence 20, 19, 18, 17, ...

$\therefore$ $a_2 - a_1 = 19 - 20 = -1$

$a_3 - a_2 = 18 - 19 = -1$

$a_4 - a_3 = 17 - 18 = -1$

i.e., $a_{k+1} - a_k$ is the same everytime.

So, the above sequence forms an AP.

Here, $a = 20$

$d = -1$

$S_n = 200$

We know that

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\Rightarrow 200 = \frac{n}{2}[2(20) + (n-1)(-1)]$$

$$\Rightarrow 200 = \frac{n}{2}[40 - n + 1]$$

$$\Rightarrow 200 = \frac{n}{2}[41 - n] \quad \Rightarrow 400 = n[41 - n]$$

$\Rightarrow n[41 - n] = 400 \quad \Rightarrow 41n - n^2 = 400$

$\Rightarrow n^2 - 41n + 400 = 0 \quad \Rightarrow n^2 - 25n - 16n + 400 = 0$

$\Rightarrow n(n-25) - 16(n-25) = 0$

$\Rightarrow (n-25)(n-16) = 0 \quad \Rightarrow n - 25 = 0$ or $n - 16 = 0$

$\Rightarrow n = 25$ or $n = 16 \quad \Rightarrow \quad n = 25, 16$

Hence, the number of rows is either 25 or 16.

Now, number of logs in row

= Number of logs in 25th row

= a_{25}

= $a + (25-1)d \qquad [\because a_n = a + (n-1)d]$

= $a + 24d$

= $20 + 24(-1) \quad \Rightarrow 20 - 24 = -4$

Which is not possible.

Therefore, $n = 16$ and

Number of log in top row

= Number of logs in 16th row

= a_{16}

= $a + (16-1)d \qquad [\because a_n = a + (n-1)d]$

= $a + 15d$

= $20 + 15(-1)$

= $20 - 15 = 5$

Hence, the 200 logs are placed in 16 rows and there are 5 logs in the top row.

EXERCISE – I UNSOLVED PROBLEMS

Q.1 If m times the mth term of an AP is equal to n times its nth term, prove that the (m + n)th term of the AP is zero.

Q.2 For an AP, show that $t_p + t_{p+2q} = 2t_{p+q}$.

Q.3 Which term of the progression 19, $18\frac{1}{5}$, $17\frac{2}{5}$, ... is the first negative term.

Q.4 Determine k so that $\frac{2}{3}$, k and $\frac{5}{8}$k are the three consecutive terms of an AP.

Q.5 Find the 27^{th} and the n^{th} terms of the sequences 5, 2, –1, –4, –7, ...

Q.6 Determine the 2nd term and rth term of an AP whose 6th term is 12 and 8th term is 22.

Q.7 Which term of the AP 5, 13, 21, ... is 181

Q.8 A sequence $\{a_n\}$ is given by the formula $a_n = 10 - 3n$. Prove that it is an AP

Q.9 The 2nd, 31st and the last term of an AP are $7\frac{3}{4}$, $\frac{1}{2}$ and $-6\frac{1}{2}$, respectively. Find the first term and the number of terms.

Q.10 The sum of three numbers in A.P. is –3, and their product is 8. Find the numbers.

Q.11 Find four numbers in A.P. whose sum is 20 and the sum of whose squares is 120.

Q.12 Find the sum of 20 terms of the A.P. 1, 4, 7, 10, ...

Q.13 Find the sum of first 30 terms of an A.P. whose second term is 2 and seventh term is 22.

Q.14 Find the sum of all natural numbers between 250 and 1000 which are exactly divisible by 3.

Q.15 How many terms of the series 54, 51, 48, ... be taken so that their sum is 513 ? Explain the double answer.

Q.16 If the sum of m terms of an A.P. is the same as the sum of its n terms, show that the sum of its (m + n) terms is zero.

Q.17 The sum of n, 2n, 3n terms of an A.P. are S_1, S_2, S_3 respectively. Prove that $S_3 = 3(S_2 - S_1)$.

Q.18 The sum of n terms of three arithmetical progression are S_1, S_2 and S_3. The first term of each is unity and the common differences are 1, 2 and 3 respectively. Prove that $S_1 + S_3 = 2S_2$.

Q.19 (i) The 4th term of an A.P. is equal to 3 times the first term and the 7th term exceeds twice the 3rd term by 1. Find the A.P.

(ii) 2nd, 31st and last term of an A.P. are $\frac{31}{4}$, $\frac{1}{2}$ and $-\frac{13}{2}$ respectively. Find the number of terms in the A.P.

Q.20 Which term of the A.P. 121, 117, 113, ..., is the first negative term ?

Q.21 In an A.P., if mth term is n and nth term is m, $m \neq n$, find
(i) the pth term (ii) (m + n)th term.

Q.22 In an A.P., the mth term is $\frac{1}{n}$ and the nth term is $\frac{1}{m}$. Find :
(i) pth term (ii) (mn)th term
(iii) sum of first mn terms.

Q.23 (i) If the nth term of a sequence is an expression of first degree in n, show that it is an A.P.
(ii) If the nth term of a sequence is given by $a_n = 2n^2 + 3$, show that it is not an A.P.

Q.24 (i) If the nth term of an A.P. is (2n + 1), find the sum of its first n terms
(ii) Find the sum of n terms of an A.P. whose kth term is 5k + 1.

Q.25 (i) How many terms of the A.P.
–6, $-\frac{11}{2}$, – 5, ... make the sum –25 ?
(ii) Solve the equation 2 + 5 + 8 +..... + x = 155.

Q.26 (i) The first term, common difference and last term of an A.P. are 12, 6 and 252 respectively. Find the sum of all the terms of this A.P.
(ii) In an A.P., if the 12th term is –13 and the sum of first four terms is 24, what is the sum of first 10 terms ?
(iii) If the sum of first 7 terms of an A.P. is 49 and that of 17 terms is 289, find the sum of first n terms.

Q.27 (i) If the sum of first n terms of an A.P. is given by $S_n = 5n^2 + 3n$, find the A.P. and its 20th term.
(ii) The sum of first n terms of an A.P. is given by $S_n = 3n^2 - 4n$. Determine the A.P. and its 12th term.
(iii) If the sum of first n terms of an A.P. is $3n^2 + 5n$ and its mth term is 164, find the value of m.

Q.28 (i) Find the sum of all two digit odd positive numbers.

(ii) Using A.P., find the sum of all 3-digit natural numbers which are divisible by 7.

(iii) Find the sum of all 3-digit natural numbers which are divisible by 13.

Q.29 (i) Find the sum of all odd integers between 2 and 100 which are divisible by 3.

(ii) Find the sum of all natural numbers from 1 to 100 which are not multiples of 4.

Q.30 (i) Find the sum of all two digit numbers which when divided by 3, yield 1 as remainder.

(ii) Find the sum of all three digit numbers which leave the remainder 1 when divided by 4.

Q.31 (i) If the third term of an A.P. is 5 and the ratio of its 6th term to the 10th term is 7 : 3, then find the sum of first 20 terms of the A.P.

(ii) The sum of first six terms of an arithmetic progression is 42. The ratio of the 10th term to the 30th term is $\frac{1}{3}$. Calculate the first and the thirteenth term.

(iii) If S_n denotes the sum of first n terms of an A.P., prove that $S_{30} = 3(S_{20} - S_{10})$.

(iv) If the ratio of the sum of first p terms and the sum of first q terms of an A.P. is $p^2 : q^2$, prove that the common difference is twice the first term.

Q.32 (i) The sum of first 15 terms of an A.P. is 105 and the sum of next 15 terms is 780. Find the first 3 terms of the A.P.

(ii) Find the common difference of an A.P. whose first term is 100 and the sum of whose first 6 terms is 5 times the sum of the next 6 terms.

Q.33 (i) The sum of three numbers in A.P. is 24 and the sum of their squares is 194. Find the numbers.

(ii) The sum of three numbers in A.P. is 30 and the ratio of first number to third number is 3 : 7. Find the numbers.

(iii) The angles of a triangle are in A.P. If the greatest angle is twice the least, find all the angles.

(iv) The sum of four numbers in A.P. is 32 and the ratio of the product of extremes to the product of means is 7 : 15. Find the numbers.

(v) The angles of a quadrilateral are in A.P. If the greatest angle is double of the smallest angle, find all the four angles.

Q.34 A manufacturer of TV sets produced 600 sets in the third year and 700 sets in the seventh year. Assuming that the production increases uniformly by a fixed number every year, find

(i) the production in the first year

(ii) the production in the 10th year

(iii) the total production in first 7 years.

Q.35 A spiral is made up of successive semicircles, with centres alternately at A and B starting with centre at A of radii 0.5 cm, 1 cm, 1.5 cm .... . What is the total length of such a spiral made up of thirteen consecutive semicircles ?

Q.36 200 logs are staked in the following manner: 20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on , in how many rows are the 200 logs placed and how many logs are there in the top row ?

Q.37 Find :

(i) 10^{th} term of the A.P. 1, 4, 7, 10, ...

(ii) 18^{th} term of the A.P. $\sqrt{2}, 3\sqrt{2}, 5\sqrt{2}$,

(iii) nth term of the A.P. 13, 8, 3, – 2, ...

(iv) 10^{th} term of the A.P. – 40, – 15, 10, 35,

(v) 8^{th} term of the A.P. 117, 104, 91, 78,

(vi) 11^{th} term of the A.P. 10.0, 10.5, 11.0, 11.5, ...

(vii) 9^{th} term of the A.P. $\frac{3}{4}, \frac{5}{4}, \frac{7}{4}, \frac{9}{4}$, ...

Q.38 (i) Which term of the A.P., 3, 8, 13, ... is 248 ?

(ii) Which term of the A.P. 84, 80, 76, ... is 0 ?

(iii) Which term of the A.P. 4, 9, 14, ... is 254

(iv) Which term is the A.P. 21, 42, 63, 84, ... is 420?

Q.39 (i) Is 68 a term of the A.P., 7, 10, 13, ... ?

(ii) Is 302 a term of the A.P. 3, 8, 13, ... ?

Q.40 (i) How many terms are there in the A.P. 7, 10, 13, ... 43 ?

(ii) How many terms are there in the A.P. $-1, \frac{5}{6}, -\frac{2}{3}, -\frac{1}{2}, ..., \frac{10}{3}$?

Q.41 The first term of an A.P. is 5, the common difference is 3 and the last term is 80 ; find the number of terms.

Q.42 The 6^{th} and 17^{th} terms of an A.P. are 19 and 41 respectively, find the 40^{th} term.

Q.43 If 9^{th} term of an A.P. is zero, prove that its 29^{th} term is double the 19^{th} term.

Q.44 If 10 times the 10^{th} term of an A.P. is equal to 15 times the 15^{th} term, show that 25^{th} term of the A.P. is zero.

Q.45 The 10^{th} and 18^{th} terms of an A.P. are 41 and 73 respectively. Find 26^{th} term.

Q.46 In a certain A.P. the 24^{th} term is twice the 10^{th} term. Prove that th 72^{nd} term is twice the 34^{th} term.

Q.47 If (m + 1)th term of an A.P. is twice the (n + 1)th term, prove that (3m + 1)th term is twice the (m + n + 1)th term.

Q.48 If the nth term of the A.P., 9, 7, 5, ... is same as the nth term of the A.P. 15, 12, 9, ... find n.

Q.49 Find the 12^{th} term from the end of the following arithmetic progressions :
(i) 3, 5, 7, 9, ... 201 (ii) 3, 8, 13, ... , 253
(iii) 1, 4, 7, 10, ... , 88

Q.50 The 4^{th} term of an A.P. is three times the first and the 7^{th} term exceeds twice the third term by 1. Find the first term and the common difference.

Q.51 Find the second term and nth term of an A.P. whose 6^{th} term is 12 and the 8^{th} term is 22.

Q.52 How many numbers of two digit are divisible by 3 ?

Q.53 An A.P. consists of 60 terms. If the first and the last terms be 7 and 125 respectively, find 32^{nd} term.

Q.54 The sum of 4^{th} and 8^{th} terms of an A.P. is 24 and the sum of the 6^{th} and 10^{th} terms is 34. Find the first term and the common difference of the A.P.

Q.55 The first term of an A.P. is 5 and its 100^{th} term is – 292. Find the 50^{th} term of this A.P.

Q.56 Find $a_{30} - a_{20}$ for the A.P. :
(i) – 9, – 14, – 19, – 24, ...
(ii) a, a + d, a + 2d, a + 3d, ...

Q.57 Write the expression $a_n - a_k$ for the AP :
a, a + d, a + 2d, ...
Hence, find the common difference of the A.P. for which
(i) 11^{th} term is 5 and 13^{th} term is 79
(ii) $a_{10} - a_5 = 200$
(iii) 20^{th} term is 10 more than the 18^{th} term.

Q.58 Find n if the given value of x is the nth term of the given A.P.
(i) 25, 50, 75, 100, ... , x = 1000
(ii) – 1, – 3, – 5, – 7 ... ; x = – 151

(iii) $5\frac{1}{2}, 11, 16\frac{1}{2}$ 22, ... ; x = 550

(iv) $1\frac{21}{11}, \frac{31}{11}, \frac{41}{11}$, ... x = $\frac{171}{11}$

Q.59 If an A.P. consists of n term a and nth term *l* show that the sum of the mth term from the beginning and the mth term from the end is (a + *l*).

Q.60 Find the arithmetic progression whose third term is 16 and seventh term exceeds its fifth term by 12.

ANSWER KEY

3. 25th term **4.** k = $\frac{16}{33}$
5. $a_{27} = -73$, $a_n = 8 - 3n$.
6. $t_2 = -8$, $t_r = 5r - 18$ **7.** 23rd term
9. First term = 8 and number of terms = 59
10. The numbers are –4, –1, 2, or 2, –1, –4.
11. The numbers are 2, 4, 6, 8 or 8, 6, 4, 2.
12. 590 **13.** 1680 **14.** 156375
15. The sum of 18 terms as well as that of 19 terms is 513. **19.** (i) 3, 5, 7, 9 (ii) 59
20. 32 nd term **21.** (i) m + n – p (ii) 0
22. (i) $\frac{p}{mn}$ (ii) 1 (iii) $\frac{1}{2}$ (mn+1)
24. **(i)** n(n+ 2) **(ii)** $\frac{n}{2}$(5n+7)
25. (i) 5 or 25 (ii) x = 29
26. (i) 54/2 (ii) 0 (iii) n^2
27. (i) 8, 18, 28, 38,...; 198(ii) –1,5,11,17, ...; 65
(iii) 27 **28.** (i) 2475 (ii) 70336 (iii) 37674
29. (i) 867 (ii) 3750 **30.** (i) 1605 (ii) 123525
31. (i) 550 (ii) 2;26
32. (i) –14, –11, –8 (ii) –10
33. (i) 7, 8, 9 (ii) 6, 10, 14 (iii) 40º, 60º, 80º
(iv) 2, 6, 10, 14 (v) 60º, 80º, 100º, 120º
34. (i) 550, (ii) 775 (iii) 4375
35. 143 cm **36.** 16 ; 5
37. (i) 28 (ii) $32\sqrt{2}$ (iii) –5n+18
(iv) 185 (v) 26 (vi) 15
(vii) $\frac{19}{4}$
38. (i) 50 (ii) 22 (iii) 51 (iv) 20^{th} term
39. (i) No (ii) No **40.** (i) 13 (ii) 27
41. 26 **42.** 87 **45.** 105 **48.** 7
49. (i) 179 (ii) 198 (iii) 55
50. First term = –3, common difference = 2
51. $a_2 = -8$, $a_n = 5n - 18$ **52.** 30
53. 69 **54.** $-\frac{1}{2}, \frac{5}{2}$ **55.** –142
56. (i) –50 (ii) 10d
57. (i) (n – k)d, 37 (ii) 40 (iii) 5
58. (i) 40 (ii) 76 (iii) 100 (iv) 17
60. 4, 10, 16, 22, ...

EXERCISE – II BOARD PROBLEMS

Q.1 The nth term (t_n) of an Arithmetic progression is given by $t_n = 7n + 1$. Find the sum of the first 30 terms of Arithmetic progression. **[Foreign-2004]**

Q.2 The 10th term of an Arithmetic progression (A.P.) is 57 and its 15th term is 87. Find the Arithmetic Progression. **[Foreign-2004]**

Q.3 If the sum of first n terms of an A.P. is given by $S_n = 3n^2 + 2n$, find the nth term of the A.P.

OR

If m times the mth term of an A.P. is equal to n times its nth term, find its (m + n)th term. **[Delhi-2004C]**

Q.4 How many terms of the A.P. 3, 5, 7,... must be taken so that the sum is 120? **[Delhi-2004C]**

Q.5 If the sum of first n terms of an A.P. is given by $s_n = 4n^2 - 3n$, find the nth term of the A.P. **[Delhi-2004C]**

Q.6 If the sum of first n terms of an A.P. is given by $S_n = 2n^2 + 5n$, find the nth term of the A.P. **[Delhi-2004C]**

Q.7 Find the sum of first 15 terms of an A.P. whose nth term is 9 – 5n.

OR

If the sum to first n terms of an A.P. is given by $S_n = 5n^2 + 3n$, find the nth term of the A.P. **[AI-2004C]**

Q.8 Find 10th term from end of the A.P. 4, 9, 14, ..., 254. **[Delhi-2005]**

Q.9 Find the number of terms of the A.P. 54, 51, 48,....so that their sum is 513.

OR

If the nth term of an A.P. is (2n + 1), find the sum of first n terms of the A.P. **[Delhi-2005]**

Q.10 Find the sum of all two digits odd positive numbers. **[AI-2005]**

Q.11 The 8th term of an Arithmetic Progression is zero. Prove that its 38th term is triple of it18th term. **[AI-2005]**

Q.12 Find the sum of all two digit positive numbers divisible by 3. **[Foreign-2005]**

Q.13 If fifth term of an A.P. is zero, show that its 33rd term is four times its 12th term. **[Foreign-2005]**

Q.14 Which term of the A.P. 5, 9, 13,... is 81? Also find the sum 5 + 9 + 13 +... + 81. **[Delhi-2005C]**

Q.15 The sum of first n terms of an A.P. is given by ($n^2 + 3n$). Find the 20th term of the progression. **[Delhi-2005C]**

Q.16 Find the sum of the first 51 terms of the A.P. whose 2nd term is 2 and 4th term is 8. **[AI-2005C]**

Q.17 The sum of the first n terms of an A.P. is given by $S_n = 3n^2 - n$. Determine the A.P. and its 25th term.

OR

The sum of three numbers in A.P. is 27 and their product is 405. Find the numbers. **[AI-2005C]**

Q.18 The 6th term of an Arithmetic Progression (A.P.) is – 10 and the 10th term is – 26. Determine the 15th term of the A.P. **[Delhi-2006]**

Q.19 Find the sum of all the natural numbers less than 100 which are divisible by 6. **[AI-2006]**

Q.20 How many terms are there in an A.P. whose first term and 6th term are – 12 and 8 respectively and sum of all its terms is 120? **[Foreign-2006]**

Q.21 Using A.P., find the sum of all 3-digit natural numbers which are multiples of 7. **[Delhi-2006C]**

Q.22 In an A.P. the sum of first n terms is $\frac{5n^2}{2} + \frac{3n}{2}$.

Find its 20th term. **[AI-2006C]**

Q.23 Find the sum of first 25 terms of an A.P. whose nth term is 1 – 4n. **[Delhi-2007]**

Q.24 Which term of the A.P. 3, 15, 27, 39, ... will be 132 more than its 54th term? **[Delhi-2007]**

Q.25 In an A.P., the sum of its first n terms is $n^2 + 2n$. Find its 18th term. **[AI-2007]**

Q.26 The first term, common difference and last term of an A.P. are 12, 6 and 252 respectively. Find the sum of all terms of this A.P. **[AI-2007]**

Q.27 The nth term of an A.P. is 7 – 4n. Find its common difference. **[Delhi-2008]**

Q.28 The sum of n terms of an A.P. is $5n^2 - 3n$. Find the A.P. Hence, find its 10th term. **[Delhi-2008]**

Q.29 The nth term of an A.P. is 6n + 2. Find it's common difference. **[Delhi-2008]**

Q.30 Find the 10th term from the end of the A.P. 8, 10, 12,...,126 **[Delhi-2008]**

Q.31 Write the next term of the A.P. $\sqrt{8}, \sqrt{18}, \sqrt{32}, ...$ **[AI-2008]**

Q.32 The sum of the 4th and 8th terms of an A.P. is 24 and the sum of the 6th and 10th terms is 44. Find the first three terms of the A.P. **[AI-2008]**

Q.33 The first term of an A.P. is p and it's common difference is q. Find it's 10th term. **[Foreign-2008]**

Q.34 For what value of n are the nth terms of two A.P.'s 63, 65, 67,...., and 3, 10, 17,.... equal?

OR

If m times the mth term of an A.P. is equal to n times it's nth term, find the (m + n)th term of the A.P. **[Foreign-2008]**

Q.35 In an A.P. the first term is 8, nth term is 33 and sum to first n terms is 123. Find n and d, the common difference. **[Foreign-2008]**

Q.36 In an A.P., the first term is 25, nth term is – 17 and sum to first n terms is 60. Find n and d, the common difference. **[Foreign-2008]**

Q.37 In an A.P., the first term is 22, nth term is – 11, and sum to first n terms is 66. Find n and d, the common difference. **[Foreign-2008]**

Q.38 For what value of p, are 2p – 1, 7 and 3p three consecutive terms of an A.P.? **[Delhi-2009]**

Q.39 If S_n, the sum of first n terms of an A.P. is given by $S_n = 3n^2 - 4n$, then find its nth term. **[Delhi-2009]**

Q.40 The sum of 4th and 8th terms of an A.P. is 24 and sum of 6th and 10th terms is 44. Find A.P. **[Delhi-2009]**

Q.41 If S_n the sum of first n terms an A.P. is given by $S_n = 5n^2 + 3n$, then find its nth term. **[Delhi-2009]**

Q.42 The sum of 5th and 9th terms of an A.P. is 72 and the sum of 7th and 12th terms is 97. Find the A.P. **[Delhi-2009]**

Q.43 If $\frac{4}{5}$, a, 2 are three consecutive terms of an A.P., then find the value of a. **[AI-2009]**

Q.44 Which term of the A.P. 3, 15, 27, 39,... will be 120 more than it's 21st term? **[AI-2009]**

Q.45 The sum of first six terms of an arithmetic progression is 42. The ratio of its 10th term to its 30th term is 1 : 3. Calculate the first and the thirteenth term of the A.P. **[AI-2009]**

Q.46 Which term of the A.P. 4, 12, 20, 28,... will be 120 more than it's 21st term? **[AI-2009]**

Q.47 For what value of k, are the numbers x, 2x + k and 3x + 6 three consecutive terms of an A.P? **[Foreign-2009]**

Q.48 The 17th term of an A.P. exceeds its 10th term by 7. Find the common difference. **[Foreign-2009]**

Q.49 If 9th term of an A.P. is zero, prove that its 29th term is double of it's 19th term. **[Foreign-2009]**

Q.50 If 5th term of an A.P. is zero, prove that it's 23rd term is three times it's 11th term. **[Foreign-2009]**

Q.51 If the 7th term of an A.P. is zero, prove that it's 27th term is five times it's 11th term. **[Foreign-2009]**

ANSWER KEY

1. 3285 **2.** 3, 9, 15, 21... **3.** 6n – 1 or 0
4. 10 terms **5.** 8n –7 **6.** 4n + 3
7. –465 or 10n –2 **8.** 209 **9.** 18 or 19 OR n (n+2)
10. 2, 475 **12.** 1, 665 **14.** 860 **15.** 42
16. 3, 774 **17.** 146 OR (3, 9, 15) or (15, 9, 3)
18. –46 **19.** 816 **20.** 12 **21.** 70, 336
22. 99 **23.** –1275 **24.** 65th term **25.** 38
26. 5412 **27.** –4 **28.** 2, 12, 22...; $a_{10} = 92$
29. 6 **30.** 108 **31.** $5\sqrt{2}$
32. –13, –8, –3 **33.** p+9q **34.** n=13 or $a_{m+n} = 0$
35. n = 6, d = 5 **36.** n = 15, d = – 3
37. n = 12, d = –3 **38.** p = 3 **39.** 6n – 7
40. –13, – 8, –3 **41.** 10n–2 **42.** 6, 11, 16, 21,....
43. $a = \frac{7}{5}$ **44.** 31st term
45. a = 2, $a_{13} = 26$ **46.** 36th term
47. k = 3 **48.** d = 1

EXERCISE – III MULTIPLE CHOICE QUESTIONS

Q.1 If 6th and 14th terms of an A.P. are 29 and 69 respectively, then its 19th term is
(A) 92 (B) 93
(C) 94 (D) 95

Q.2 If the sum of first n terms of an A.P. is $2n^2 + 5n$ then its nth term is
(A) 4n + 3 (B) 4n – 3
(C) 3n – 4 (D) 3n + 4

Q.3 If the sum of first m terms of an A.P. is n and the sum of its first n terms is m, then the sum of its first (m + n) terms will be
(A) m + n (B) m – n
(C) –(m + n) (D) 0

Q.4 The tenth term from the end of the A.P. 4, 9, 14, ..., 254 is
(A) 214 (B) 209
(C) 208 (D) 213

Q.5 If the nth term of the sequence –1, 4, 9, 14, ... is 129, then n is equal to
(A) 25 (B) 26
(C) 27 (D) 28

Q.6 The first negative term of the sequence 20, $\frac{77}{4}, \frac{37}{2}, \frac{71}{4}$, ... is
(A) 27th (B) 28th
(C) 29th (D) 26th

Q.7 If 8 times the eighth term of an A.P. is equal to 7 times its seventh term, then its 15th term is
(A) 15 (B) 22
(C) 0 (D) 23

Q.8 The the p^{th} term of an A.P. is q and its q^{th} term is p, then its nth term is
(A) p + q – 1 (B) p + q + 1
(C) p + q (D) p + q – n

Q.9 If the nth term of an A.P. is 2n + 1, then the sum of its first n terms is
(A) $(2n + 1)^2$ (B) $n^2 + 2$
(C) $n^2 + 2n$ (D) $n^2 + 4$

Q.10 If the sum of first n terms of an A.P. is $\frac{n}{2}$ (3n + 5), then its 25th term is
(A) 76 (B) 73
(C) 79 (D) 70

Q.11 If the sum of first 9 terms of an A.P. is equal to sum of its first 11 terms, then sum of its first 20 terms is
(A) 0 (B) 20
(C) 31 (D) 29

Q.12 If the sum of first k terms of an A.P. is $3k^2 - k$ and its common difference is 6, then its first term is
(A) 1 (B) 2
(C) 3 (D) 4

Q.13 If first and last terms of an A.P. are 1 and 11 respectively and the sum of its terms is 36, then the number of terms in this A.P. is
(A) 8 (B) 7
(C) 6 (D) 5

Q.14 If the sum of first 6 terms of an A.P. is 9 and the sum of its first 9 terms is 6, then sum of its first 15 terms is
(A) 0 (B) 15
(C) –20 (D) –15

Q.15 The number of terms of the A.P. –5, –1, 3, 7, 11, ... to be taken so that the sum is 400, is
(A) 7 (B) 12
(C) 14 (D) 16

Q.16 If the first term of an A.P. is 2 and common differecne is 4, then the sum of its first 40 terms is
(A) 200 (B) 1600
(C) 3200 (D) 2800

Q.17 The list of numbers –6, –3, 0, 3, ...
(A) does not form an A.P.
(B) is an A.P. with common difference –9
(C) is an A.P. with common difference –3
(D) is an A.P. with common difference 3

Q.18 Which of the following lists of numbers does not form an A.P. ?

(A) –2, 2, –2, 2, ...

(B) $-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}$, ...

(C) $3, 3+\sqrt{2}, 3+2\sqrt{2}, 3+3\sqrt{2}$, ...

(D) $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}$ .....

Q.19 Which of the following lists of numbers does not form an A.P. ?
(A) –1.2, –3.2, –5.2, –7.2 .....
(B) 0, –4, –8, –12, ...
(C) $1^2, 3^2, 5^2, 7^2$, ...
(D) $1^2, 5^2, 7^2$, 73, ...

Q.20 30th term of the A.P. 10, 7, 4, ... is
(A) 87 (B) 77
(C) –77 (D) –87

Q.21 11th term of the A.P. $-3, -\frac{1}{2}$, 2, ... is
(A) 28 (B) 22
(C) –38 (D) $-48\frac{1}{2}$

Q.22 If the first two terms of an A.P. are 3 and –2 respectively, then its 20th term is
(A) 98 (B) 22
(C) –16 (D) –92

Q.23 In an A.P., if 3 times its 3rd term is equal to 7 times its 7th term, then its 10th term is equal to
(A) 0 (B) 3
(C) 7 (D) 10

Q.24 The 15th term from the end of the A.P. 7, 10, 13, ..., 130, is
(A) 49 (B) 85
(C) 88 (D) 110

Q.25 In an A.P., if $a_{20} - a_{12} = -24$, then its common difference is
(A) 3 (B) –3
(C) 8 (D) –8

Q.26 If the common difference of an A.P. is –2, then $a_{30} - a_{12}$ is equal to
(A) 24 (B) 36
(C) –36 (D) –60

Q.27 If k – 1, k + 1 and 2k + 3 are in A.P., then the value of k is
(A) –2 (B) 0
(C) 2 (D) 4

Q.28 If k + 2, 4k – 6 and 3k – 2 are three consecutive terms of an A.P., then the value of k is
(A) 3 (B) –3
(C) 4 (D) –4

Q.29 If the last term of the A.P. 5, 3, 1, –1, ... is –41, then the A.P. consists of
(A) 46 terms (B) 25 terms
(C) 24 terms (D) 23 terms

Q.30 First term of an A.P. is –5 where as the first term of another A.P. is 16 If both the A.P.'s have the same common difference, then the difference between 25th term of 2nd A.P. and the 25th term of 1st A.P. is
(A) –5 (B) 16
(C) 21 (D) –21

Q.31 The number of two digit numbers which are divisible by 3 is
(A) 33 (B) 31
(C) 30 (D) 29

Q.32 The number of multiples of 4 that lie between 10 and 250 is
(A) 62 (B) 60
(C) 59 (D) 55

Q.33 The sum of 25 terms of the A.P. $-\frac{2}{3}, -\frac{2}{3}, -\frac{2}{3}, \ldots\ldots$
(A) 0 (B) $-\frac{2}{3}$
(C) $-\frac{50}{3}$ (D) –50

Q.34 The sum of 22 terms of the A.P. 8, 3, –2, ... is
(A) –979 (B) – 379
(C) 979 (D) 1331

Q.35 The sum of first 10 even whole numbers is
(A) 110 (B) 90
(C) 55 (D) 45

Q.36 The sum of numbers between 50 and 100 which are divisible by 7 is
(A) 616 (B) 593
(C) 539 (D) 462

Q.37 In an A.P., if a = –5, ℓ = 21 and S = 200, then n is equal to
(A) 50 (B) 40
(C) 32 (D) 25

Q.38 In an A.P., if a = 3, and $S_8 = 192$, then d is
(A) 8 (B) 7
(C) 6 (D) 4

Q.39 If the nth term of an A.P. is given by $a_n = 5n - 3$, then the sum of first 10 term is
(A) 225 (B) 245
(C) 255 (D) 270

Q.40 How many two digit number are there which are divisible by 7 ?
(A) 13 (B) 14
(C) 15 (D) None

Q.41 How many numbers are there between 103 and 750 which are divisible by 6 ?
(A) 125 (B) 108
(C) 107 (D) 113

Q.42 The sum of first 60 natural numbers is
(A) 1830 (B) 1640
(C) 3660 (D) 1770

Q.43 The sum of all 2 digit numbers is –
(A) 4750 (B) 4895
(C) 3776 (D) 4680

Q.44 23^{rd} term of the A.P. 7, 5, 3, 1, ... is –
(A) 51 (B) 37
(C) –37 (D) –51

Q.45 If (k + 1), 3k and (4k + 2) be any three consecutive terms of an A.P., then the value of k is –
(A) 3 (B) 0
(C) 1 (D) 2

Q.46 Which term of the A.P. 5, 8, 11, 24 .... is 320 ?
(A) 104^{th} (B) 105^{th}
(C) 106^{th} (D) 64^{th}

Q.47 The 5^{th} and 13^{th} terms of an A.P. are 5 and –3 respectively. The first term of the A.P. is –
(A) 1 (B) 14
(C) –15 (D) 2

Q.48 Which term of the A.P. 64, 60, 56, 52, ...is zero?
(A) 16^{th} (B) 17^{th}
(C) 14^{th} (D) 15^{th}

Q.49 The n^{th} term of an A.P. is (3n + 5). Its 7^{th} term is –
(A) 26
(B) (3n–2)
(C) 3n + 12
(D) cannot be determined

Q.50 The sides of a right angle triangle are in A.P. The ratio of side is –
(A) 1 : 2 : 3 (B) 2 : 3 : 4
(C) 3 : 4 : 5 (D) 5 : 8 : 3

Q.51 The sum of 1, 3, 5, 7, 9, ... upto 20 terms is–
(A) 400 (B) 563
(C) 472 (D) 264

Q.52 The sum of the series 5 + 13 + 21 + ... + 181 is–
(A) 2139 (B) 2476
(C) 2219 (D) 2337

Q.53 The sum of all odd numbers between 100 and 200 is –
(A) 6200 (B) 6500
(C) 7500 (D) 3750

Q.54 The sum of all positive integral multiples of 5 less than 100 is –
(A) 950 (B) 1230
(C) 760 (D) 875

Q.55 The sum of all even natural numbers less than 100 is –
(A) 2450 (B) 2272
(C) 2352 (D) 2468

Q.56 Arithmetic mean between 14 and 18 is
(A) 16 (B) 15
(C) 17 (D) 32

Q.57 If 4, A_1, A_2, A_3, 28 are in A.P., then the value of A_3 is –
(A) 23
(B) 22
(C) 19
(D) cannot be determined

Q.58 How many terms of the A.P. 3, 6, 9, 12, 15, ... must be taken to make the sum 108 ?
(A) 6 (B) 7
(C) 8 (D) 36

Q.59 If k – 2, 2k + 1 and 6k + 3 are in G.P., the value of k is –
(A) 7 (B) 0
(C) 3 (D) –2

Q.60 The 8^{th} term of the G.P. 2, 6, 18, 54, ... is –
(A) 2187 (B) 4374
(C) 1098 (D) 3682

Q.61 If n^{th} term of a G.P. is 2^n, then its 6^{th} term is–
(A) 2^6 (B) 2^5
(C) $2^6/6$ (D) $(2^6 \times 6)$

Q.62 Which term of the G.P. 5, 10, 20, 40, ... is 1280–
(A) 10^{th} (B) 9^{th}
(C) 8^{th} (D) None of these

Q.63 If a, b, c are in G.P. and $a^{1/x} = b^{1/y} = c^{1/z}$, then x, y, z are in –
(A) A.P. (B) G.P.
(C) H.P. (D) None of these

Q.64 If a, b, c are in G.P., then log a, log b, log c are in –
(A) A.P. (B) G.P.
(C) H.P. (D) None of these

Q.65 If $\frac{1}{3}$, y_1, y_2, 9 are in G.P., the value of y_2 is–
(A) 3 (B) 6
(C) 1
(D) Cannot be determined

Q.66 The A.M. of two number is 34 and their G.M. is 16. The number are –
(A) 60, 8 (B) 64, 4
(C) 56, 12 (D) 52, 16

Q.67 Relation between A.M. and G.M. is –
(A) A.M. $\leq$ G.M.
(B) A.M. $\geq$ G.M.
(C) A.M. = $\frac{3}{4}$ G.M.
(D) None of these

Q.68 If a, G, b are in G.P., then –
(A) G = ab (B) G^2 = ab
(C) G = $\frac{1}{2}$ ab (D) G = $\frac{1}{2}$ (a + b)

Q.69 $6^{1/2} . 6^{1/4} . 6^{1/8}$ ..... ∞ = ?
(A) 6 (B) ∞
(C) 216 (D) 36

Q.70 $1 - \frac{1}{3} + \frac{1}{3^2} - \frac{1}{3^3}$ + ... ∞ = ?
(A) $\frac{1}{2}$ (B) $\frac{1}{6}$ (C) $\frac{3}{4}$ (D) $\frac{4}{9}$

Q.71 The sum of 6 terms of the G.P.
1, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, ...is
(A) $\frac{93}{64}$ (B) $\frac{63}{64}$ (C) $\frac{1023}{512}$ (D) $\frac{19}{36}$

Q.72 The 6^{th} and 8^{th} terms of an A.P. are 12 and 22 respectively, Its 2^{nd} term is –
(A) 8 (B) –8
(C) 6 (D) –3

Q.73 The 3^{rd} and 5^{th} terms of a G.P. are 12 and 48. Its second term is –
(A) 6 (B) 4
(C) 8 (D) 1/2

Q.74 The fraction equivalent to $\frac{2}{5}\%$ is:
(A) 1/40 (B) 1/125
(C) 1/250 (D) 1/500

Q.75 0.025 in terms of rate per cent is:
(A) 0.25% (B) 2.5%
(C) 25% (D) 37%

Q.76 5% of 10% of Rs. 175 is equal to:
(A) Rs. 8.75 (B) Rs. 0.50
(C) Rs. 0.875 (D) Rs. 17.50

Q.77 If 40% of 40% of x = 40, then the value of x is:
(A) 100 (B) 400
(C) 250 (D) 1000

Q.78 In an examination 93% of students passed and 259 failed the total number of student is:
(A) 3700 (B) 500
(C) 3950 (D) none of these

Q.79 A man received 10% increase in his salary. His new salary Rs. 5060 his original salary was:
(A) Rs. 4362 (B) Rs. 4400
(C) Rs. 4600 (D) Rs. 4560

Q.80 Gun powder contains 75% nitrate, 10% of sulphur and the rest is charcoal. The quantity of charcoal in 24 kg of gun powder is:
(A) 3.6 kg (B) 2.4 kg
(C) 18 kg (D) none of these

Q.81 In an examination, 52% of the candidates failed is English 42% failed in Mathematics and 17% failed in both. The number of those who have passed in both the subjects is:
(A) 23% (B) 35%
(C) 25% (D) 40%

Q.82 If x is 90% of y what per cent of x is y?
(A) 90 (B) 190
(C) 101.1 (D) 111.1

Q.83 The price of cooking oil has increased by 25%. The percentage of reduction that a family should effect in the use of cooking oil so as not to increase the expenditure on this account is:
(A) 15% (B) 20%
(C) 25% (D) 30%

Q.84 1 litre of water is added to 5 litres of a 20% solution of alcohol in water the strength of alcohol is:
(A) $12\frac{1}{2}\%$ (B) $16\frac{1}{2}\%$
(C) 24% (D) 16%

Q.85 A man spends 75% of his income. His income is increased by 20% and he increased his expenditure by 10%. His savings are increased by:
(A) 10% (B) 25%
(C) $37\frac{1}{2}\%$ (D) 50%

Q.86 The length and breadth of a square are increased by 30% and 20% respectively. The area of the rectangle so formed exceeds the area of the square by:
(A) 20% (B) 36%
(C) 50% (D) 56%

Q.87 If 10% of in is the same as 20% of n then m:n is equal to:
(A) 1:2 (B) 2:1
(C) 5:1 (D) 10:1

Q.88 If A:B = 2:3 and B:C = 4:5, then C:A is equal to:
(A) 15 : 8 (B) 12 : 10
(C) 8 : 5 (D) 8 : 15

Q.89 Rs. 680 has been divided among A, B, C such that A gets 2/3 of what B gets and B gets 1/4 of what C gets. Then B's share is:
(A) Rs. 60 (B) Rs. 80
(C) Rs. 120 (D) Rs. 160

Q.90 Rs. 600 has been divided among A, B and C is such away that Rs. 40 more than 2/7 of A's share, Rs. 20 more than 2/7 of B's share is to more than 9/17 of C's share are all equal. A's share is:
(A) Rs. 280 (B) Rs. 170
(C) Rs. 150 (D) Rs. 200

Q.91 A bag contains Rs. 102 in the form of rupee, 50 paise and 10 paise coins in ratio 3:4. The number of 10 paise coins is:
(A) 60 (B) 80
(C) 170 (D) 340

Q.92 729 ml of a mixture contains milk and water in the ratio 7:2. How much more water is to be added to get a new mixture containing milk and water in the ratio 7:3?
(A) 600 ml b) 710 ml
(C) 520 ml (D) none of these

Q.93 A certain amount was divided between Kavita and Reena in the ratio 4:3. If Reena's share was Rs. 2400, the amount is:
(A) Rs. 5600 (B) Rs. 3200
(C) Rs. 9600 (D) none of these

Q.94 If the weight of a 13 metre long iron rod be 23.4 kg. The weight of 6 metre long of such rod will be:
(A) 7.2 kg (B) 12.4 kg
(C) 10.8 kg (D) 18 kg

Q.95 A and B enter into partnership investing Rs. 12000 and Rs. 16000 respectively. After 8 month, C also a joins the business with a capital of Rs. 15000. The share of C in a profit of Rs. 45600 after 2 years will be:
(A) Rs. 12000 (B) Rs. 14400
(C) Rs. 19200 (D) Rs. 21200

Q.96 Suresh invested Rs. 12,000 in a shop and Dinesh joined him after 4 month by investing Rs. 7000, if the net profit after one year be Rs. 13,300 Dinesh's share in the profit:
(A) Rs. 3,724 (B) Rs. 4,900
(C) Rs. 9,400 (D) none of these

Q.97 A and B enter into partnership investing Rs. 12000 and Rs. 16000 respectively. After 8 months, C also joins the business with a capital of Rs. 45600 after 2 year will be:
(A) Rs. 12,000 (B) Rs. 14,400
(C) Rs. 19,200 (D) Rs. 21,200

Q.98 A, B and C enter into partnership by making investments in the ratio 3:5:7. After a year, C invests another Rs. 3,37,600 while A with draws Rs. 45,600. The ratio of investments, then changes to 24:59:167, How much did A invest initially?
(A) Rs. 45,600 (B) Rs. 96,000
(C) Rs. 1,41,600 (D) none of these

Q.99 The average of 25 results is 18; that of first 12 is 14 and of the last 12 is 17 their thirteenth is:
(A) 78 (B) 85
(C) 28 (D) 72

Q.100 The sum of three numbers is 98, if the ratio between first and second be 2:3 and that between second and third be 5:8, then the second number is:
(A) 30 (B) 20
(C) 58 (D) 48

Q.101 The average age of 11 players of a cricket team is decreased by 2 months when two of them ages 17 years and 20 years are:
(A) 18 year 3 month
(B) 17 year 1 month
(C) 17 year 7 month
(D) 17 year 11 month

Q.102 The average temperature of first 3 days is 27°C and the next 3 days is 29°C. If the averages of the whole week is 28.5°C. The temperature of the last day is:
(A) 31.5° (B) 10.5°
(C) 21° (D) 42°

Q.103 A cricketer scored 180 runs in the first test and 258 runs in the second test match. How many runs should he score in the third test his average score in the three tests would be 230 runs?
(A) 219 (B) 252
(C) 334 (D) none of these

Q.104 A ship sails out to a mark at the rate of 15 km/hr and sails back at the rate of 10 km/hr. The average rate of sailing is:
(A) 12.5 km/hr (B) 12 km/hr
(C) 25 km/hr (D) 5 km/hr.

Q.105 The mean temperature of Monday to Wednesday was 37°C and Tuesday to Thursday was 34ºC. If the temperature on Thursday was 4/5th that of Monday, the temperature on Thursday was:
(A) 34°C (B) 35.5°C
(C) 36°C (D) 36.5°C

Q.106 If 4 examiners can examine a certain number of answer books in 8 days by working 5 hours a day; for how many hours a day would 2 examiners have to work in order to examine twice the number of answer books in 20 days?
(A) 6 hour (B) 8 hour
(C) 9 hour (D) 7.30 hour

Q.107 120 men had provisions for 200 days. After five days 30 men dead due to an epedimic. The remaining food will last 10:
(A) 150 day (B) $146\frac{1}{4}$ day
(C) 245 day (D) 260 day

Q.108 A gourison had provisions for certain number of after 10 days 1/5th of the men and-it is found that the provisions will now last just as long as before how long was what?
(A) 35 day (B) 15 day
(C) 25 day (D) 50 day

Q.109 If 9 men working $7\frac{1}{2}$ hours a day can finish a work is 20 days, then how many days will be taken by 12 men, working 6 hours a day to finish the work, it being given that 3 men of letter type work as much as 2 men of the former type in the same time ?
(A) $12\frac{1}{2}$ (B) 13
(C) $9\frac{1}{2}$ (D) 11

Q.110 If 3 men or 6 boys can do a piece of work in 10 days working 7 hours a day, how many days will it take to complete a work twice as large with 6 men and 2 boys working together for 8 hours a day?

(A) $7\frac{1}{2}$ (B) $8\frac{1}{2}$

(C) 9 day (D) 6 day

Q.111 2 men and 7 boys can do a piece of work in 14 days; 3 men and 8 boys can do the same in 11 days, 8 men and 6 boys can do 3 times the amount of their work is:

(A) 21 day (B) 18 day

(C) 24 day (D) 36 day

Q.112 Two partners invested Rs. 1250 and Rs. 850 respectively in a business, they distribute 60% of the profit equally and decide to distribute the remaining 40% as the interest on their capitals. If one partner received Rs. 30 more than the other. The total profit is:

(A) Rs. 840 (B) Rs. 393.75

(C) Rs. 590 (D) none of these

Q.113 A and B enter into partnership a supplies whole of the capital amount to Rs. 45000 with the condition that the profits are to be equally divided and that B pay A interest on half the capital at 10% per annum but receives Rs. 120 per month for carrying on the concern. Find their total yearly profit when B's income is one half of A's income:

(A) Rs. 3060 (B) Rs. 980

(C) Rs. 2750 (D) Rs. 1140

Q.114 A boy goes to school with the speed of 3 km per hour and returns with a speed of 2 km/hr. If he takes 5 hours in all the distance in kms between the village and the school is:

(A) 6 (B) 7

(C) 8 (D) 9

Q.115 A thief steals a car at 1.30 pm and drives it at 40 km per hour and covered distance at 2 p.m. and the owner sets off in another car at 50 km per hour, he will overtake the thief at:

(A) 4 pm (B) 3.30 pm

(C) 6 pm (D) 4.30 pm

Q.116 A certain distance is covered at a certain speed. If half their distance is covered in doubled the time. The ratio of the two speed is:

(A) 4 : 1 (B) 1 : 4

(C) 2 : 1 (D) 1 : 2

Q.117 Suresh started cycling along the boundaries of a square field from corner point A. After half an hour he reached the corner point C diagonally opposite A. If his speed was 8 km/hr. The area of the field in square km is:

(A) 64 square km (B) 8 square km

(C) 4 square km (D) none of these

Q.118 Suresh has cover a distance of 6 km is 45 minutes. If he covers one half of the distance 2 in -rd time, what should the speed in km/hr to cover the remain distance in is:

(A) 12 (B) 16

(C) 3 (D) 8

Q.119 A man is standing on a railway bridge which is 50 metre long. He finds that a train 1 crosses the bridge in 4— second but himself in 2 second. The speed of a train is:

(A) 40 km/hr (B) 50 km/hr

(C) 72 km/hr (D) none of these

Q.120 A man sitting in the train which is traveling at the rate of 50 km/hr observes that it takes 9 seconds for a goods trains traveling in the opposite direction to pass him. If the goods train is 187.5 m long. The speed of a train is:

(A) 75 km/hr (B) 45 km/hr

(C) 25 km/hr (D) none of these

Q.121 A tram 75 metres long overtook a person who was walking at the rate of 6 km per hour and passed him in $7\frac{1}{2}$ seconds. Subsequently overtook a second person and 3 passed him in $6\frac{3}{4}$ seconds. At what rate was the second person traveling?

(A) $2\frac{1}{3}$ km/hr (B) 2 km/hr

(C) $2\frac{3}{4}$ km/hr (D) none of these

Q.122 Two trains are running on parallel lines in the same direction at a speed of 50 km/hour and 30 km/hour respectively. The faster train crosses a man in slower train in 18 seconds. The length of the faster train is:

(A) 98 m (B) 170 m

(C) 100 m (D) 85 m

Q.123 Two trains of equal length are running on parallel lines in the same direction at the rate of 46 km/hr and 36 km/hr. The faster train passes the slower train in 36 seconds, the length of each train is:

(A) 50 m (B) 80 m

(C) 72 m (D) none of these

Q.124 A man can row 4.5 km/hr in still water and he finds that it takes him twice as long to low down the river. The rate of stream:
(A) 2.5 km/hr (B) 1.5 km/hr
(C) 3 km/hr (D) 1.75 km/hr

Q.125 The current of a stream runs at the rate of 2 km/hr. A motor boat goes 10 km upstream and back again to the starting point in 55 minutes. The speed of the motor boat in still water:
(A) 22 km/hr (B) 18 km/hr
(C) 65 km/hr (D) none of these

Q.126 A man can row 30 km upstream and 44 km downstream in 10 hours while he can row 40 km upstream and 55 km downstream in 13 hours. The rate of the current and the speed of the man in still water:
(A) 8 km/hr and 5 km/hr
(B) 3 km/hr and 8 km/hr
(C) 12 km/hr and 3 km/hr
(D) 2.5 km/hr and 5 km/hr

Q.126 A man sold two horses for Rs. 924 each. On one he gain 12% and another he loses 12%. How much does he gain or lose in the whole transaction?
(A) 1.84% (B) 2.12%
(C) 2.25% (D) 1.44%

Q.127 A manufacturer sells goods to an agent at a profit of 20%. The agent's whole sale price to a shopkeeper is at a profit of 10% and the shopkeeper retails his goods at a profit of 12.5%. The manufacturing cost of goods bought from the shop for Rs. 14.85 is:
(A) Rs. 132 (B) Rs. 10
(C) Rs. 100 (D) Rs. 297

Q.128 A tradesman marks his goods at such a price that after allowing a discount of 1 12.5% for cash, he makes a profit of 20%. What is the marked price of an article which costs him Rs.210?
(A) Rs. 144 (B) Rs. 288
(C) Rs. 100 (D) none of these

Q.129 A tradesman bought 500 metres of electric wire at Rs. 75 per metre. He sold 60% of it at a profit of 8%. At what gain per cent should he sell the remainder so as to gain 12% on the whole transaction?
(A) 18% (B) 25%
(C) 60% (D) 12%

Q.130 A merchant buys 200 kg of rice at Rs. 7.25 per kg, 400 kg of rice at Rs. 5.75 per kg. He mixes them and sells one third of mixture at Rs. 6 per kg. At what rate should he sell the remaining mixture so that he may earn a profit of 20% on the whole out lay?
(A) Rs.28.25 per kg (B) Rs.13.25 per kg
(C) Rs.8.25 per kg (D) none of these

Q.131 A man possessing Rs. 8400 lent a part of it at 8% simple interest and a remaining at $6\frac{2}{3}$th part simple interest. His total income after $1\frac{1}{2}$ year was Rs. 882. Find the sum lent at different rates:
(A) Rs.6330 (B) Rs.1012
(C) Rs.7760 (D) Rs.2100

Q.132 A lamp of 2 metals weight 189 is worth Rs. 87. But if there weight is be interchanges it would be worth Rs. 78.60. If the price of one metal be Rs. 65.70 per gram. Find the weight of another metal in mixture:
(A) 9 gm (B) 7 gm
(C) 8 gm (D) 3 gm

Q.133 Two Vessels A and B containing milk and water mixed in the ratio 5:2 and 8:5 respectively. Find the ratio in which these mixture are to be mixed to get a new mixture containing milk and water in the ratio 9:4?
(A) 6 : 2 (B) 7 : 3
(C) 7 : 2 (D) 6 : 1

Q.134 A can contains a mixture of two liquids A and B in proportion 7 : 5 when 9 litre of mixture are drawn off and the can is filled with B, the proportion of A and B becomes 7 : 9. How many litre of liquid A was contained by the can initially?
(A) 25 (B) 10
(C) 20 (D) 21

Q.135 A certain sum of money amounts to Rs. 678 in 2 years and to Rs. 736.50 in 3.5 years. The rate of interests is:
(A) $2\frac{2}{3}\%$ (B) $6\frac{1}{2}\%$
(C) $1\frac{1}{2}\%$ (D) none of these

Q.136 A merchant borrowed Rs. 2500 from the money lenders. For one loan he paid 12% per annum and for the other 14% per annum. The total interest paid for one year was Rs. 326. How much did he borrow at each rate?
(A) 1200 and 1300 (B) 130 and 175
(C) 225 and 350 (D) none of these

Q.137 Simple interest on a certain sum is 9/16 of the sum. The rate per cent and time if both are equal is:
(A) $3\frac{2}{5}$ year (B) $2\frac{1}{5}$ year
(C) $6\frac{2}{5}$ year (D) $7\frac{1}{2}$ year

Q.138 A sum was put at simple interest at a certain rate for 2 years. Had it been put at 3% higher rate, it would have fetched Rs. 72 more find the sum:
(A) Rs. 1000 (B) Rs. 1200
(C) Rs. 1500 (D) none of these

Q.139 What annual installment will discharge a depth of Rs. 2710 due in 4 years at 7% simple interest?
(A) Rs. 1000 (B) Rs. 225
(C) Rs. 500 (D) Rs. 150

Q.140 A man promises his wife a birthday present going her each year a number of rupees equal to the number of years in her age. If her birthday falls on August 8th, what sum must be placed at simple interest at 7% on January, 1st before she is 42, in order to raise the sum?
(A) Rs. 1325 (B) Rs. 1000
(C) Rs. 1600 (D) Rs. 1225

Q.141 If the compound interest on a certain sum of money for $2\frac{1}{2}$ years at 5% per annum be Rs.104.05, what would be the simple interest?
(A) Rs. 215.75 (B) Rs. 125.75
(C) Rs. 100 (D) Rs. 225

Q.142 The difference between the compound and the simple interest on a certain sum at $7\frac{1}{2}\%$ per annum for 3 years is Rs. 110.70 Find the sum:
(A) Rs. 6400 (B) Rs. 1200
(C) Rs. 320050 (D) none of these

Q.143 If the cost of articles is Rs. 650, then the cost of 30 articles is:
(A) Rs. 450 (B) Rs. 975
(C) Rs. 800 (D) Rs. 895

Q.144 Rano runs 1000 metres while Rupa runs 800 metres, Rupa runs 300 metres while Astha runs 400 metres. How many metres can Astha run while Rano runs 600 metres:
(A) 640 m (B) 680 m
(C) 720 m (D) 730 m

Q.145 A contract is to be completed in 56 days and 104 men were set to work each working 8 hours a day. After 30 days, 2/5 of the work is completed. How many additional men be employed, so that the work may be completed in time each man row working hour a day?
(A) 47 (B) 49
(C) 48 (D) 56

Q.146 A rope makes 140 rounds of the circumference of a cylinder whose radius of the base is 14 cms. How many times can it go round a cylinder with radius 20 cm?
(A) 98 (B) 96
(C) 90 (D) 100

Q.147 The average $31\frac{1}{2}$, $32\frac{1}{4}$, $31\frac{2}{3}$ and $33\frac{3}{4}$ is:
(A) $42\frac{7}{23}$ (B) $32\frac{7}{24}$
(C) $17\frac{23}{24}$ (D) $31\frac{1}{3}$

Q.148 Some men visited a hotel, 8 of them spent Rs. 4 each over their meal and the 9th spent Rs. 2 more than the average of all the nine. The total money spent by them on their meals:
(A) Rs. 40 (B) Rs. 38.25
(C) Rs. 38 (D) Rs. 38.50

Q.149 Of the three numbers the first is twice the second and it half the third. If the average of the number is 50. The three numbers in order are:
(A) 48, 24, 96 (B) 48, 36, 96
(C) 48, 12, 14 (D) 24, 12, 48

Q.150 A company makes a profit of Rs. 9,00,000 20% of which is paid as taxes. If the rest is divided among three partners. P, Q and R in the ratio of 1 : $\frac{1}{2}$: 2 then the shares of P, Q and R in rupees are respectively:
(A) 2,40,000; 3,20,000; 1,60,000
(B) 3,20,000; 2,40,000; 1,60,000
(C) 1,60,000; 3,20,000; 2,40,000
(D) 1,60,000; 2,40,000; 3,20,000

Q.151 1/3 and 1/4 parts of two cans of equal volume are filled with milk. The cans are filled to capacity with water and the mixture of the two cans are poured in a big pot. The ratio of milk and water in the new pot is:
(A) 3 : 16 (B) 7 : 17
(C) 9 : 72 (D) 11 : 27

Q.152 If 3 persons can do 3 times of a particular work in 3 days then 7 persons can do 7 times of that work in:
(A) 7 day (B) 6 day
(C) 4 day (D) 3 day

Q.153 In an examination, 80% of the examines passed in English, 85% passed in Mathematics and 75% passed in both English and Mathematics. If 45 examines failed in both English and Mathematics, the number of examines were:
(A) 450 (B) 400
(C) 250 (D) 300

Q.154 Two successive discounts of 10% and 5% are equal to a single discount of:
(A) 7% (B) 12.5%
(C) 14.5% (D) 15%

Q.155 In an innings Sachin's score was 2/7th of the total and Ganguly's 2/7th of the remainder. If Sachin scored 32 runs more than Ganguly, what was the total score?
(A) 350 (B) 364
(C) 378 (D) 392

Q.156 x% of y is y% of:
(A) x (B) y/100
(C) x/100 (D) none of these

Q.157 1 added 1 litre of water to 5 litres of a 20% solution of alcohol in water. The strength of alcohol is:
(A) 6% (B) 5%
(C) 12.5% (D) 16.66%

Q.158 The captain of a cricket team of 11 players in 25 years older and the wicketkeeper is 3 years older. If the ages of these two are excluded, the average age of the remaining player is 1 year less than the average are of the whole team. The average of the whole team is:
(A) 21.5 year (B) 22 year
(C) 22.5 year (D) 23 year

Q.159 Excluding stoppages, the speed of a train is 45 km/h and including stoppages, it is 36 km/h. For how many minutes does train stop per hour?
(A) 10 (B) 12
(C) 15 (D) 18

Q.160 The charge for sending a telegram is constant for the first 10 or less words and an amount proportional to the number of words exceeding 10. If the charge for sending a 15 words telegram is Rs. 3, and that for a 20 words is Rs. 4.25, how much would it cost to send a 35 words telegram?
(A) Rs. 7.00 (B) Rs. 7.25
(C) Rs. 8.00 (D) Rs. 7.75

Q.161 Shri ram purchases 6 tables and 8 chairs for Rs. 1350, he then sells then all such that he receives same x% profit and x% loss on chairs and tables respectively and over all he suffers loss of 30 then x is equal to:
(A) 20% (B) 15%
(C) 10% (D) data insufficient

Q.162 5 boys, 8 girls and 3 teachers are to be selected for a group formation. What is the maximum number of distinct group that can be formed if all the groups have atleast one member of each kind and none of the group have all the members of the same kind?
(A) 18 (B) 104
(C) 11430 (D) 45720

Q.163 A machinery costs Rs. 5,00,000. It depreciates by 25% in the first year, 20% in the second, 15%, in the third and 10% in the fourth year. What will be its value at the beginning of the fifth year?
(A) 2,19,500 (B) 2,29,000
(C) 19,000 (D) 2,29,500

Q.164 A and B made a joint stock of Rs. 2700 by which they gained Rs. 150 of which A had as his share Rs. 10 more than B. If the profit was shared in the ratio of their contributions. What did A contribution to the stock?
(A) Rs. 1260 (B) Rs. 1350
(C) Rs. 1440 (D) Rs. 1500

Q.165 A man is running at the speed of 45 km/h just then a train travelling in the same direction, at the speed of 60 km/hr crosses the man. If the train is 200 m long then how much time will the train take to cross the man?
(A) 45 second (B) 52 second
(C) 48 second (D) 47.5 second

Q.166 Ram travelled 1440 km by train which formed 2/5th of his journey. He travelled one third of the whole trip by bus and the rest by air, what was the distance travelled by air?
(A) 860 km (B) 1560 km
(C) 960 km (D) 460 km

Q.167 What is the market value of 10.5% of stock whose per value is Rs. 100 and that gives an income of Rs. 1260 on investing Rs. 9000?
(A) Rs. 125 (B) Rs. 75
(C) Rs. 80 (D) Rs. 90

Q.168 What is the least number of four digits when increased by 7 divisible by 35, 48 and 56?
(A) 1480 (B) 1473
(C) 1487 (D) 1673

Q.169 A money lender lent a total amount of Rs. 47,000 to A, B and C at 5%, 3% and 4% per annum respect as rate of interest. If the same amount of interest is paid by three persons at the end of 7 year, 10 year and 5 year respectively. The amount borrowed by C is:
(A) Rs. 14000 (B) Rs. 12000
(C) Rs. 15000 (D) Rs. 21000

Q.170 3 women and 4 boys can finish a work in $2\frac{1}{2}$ days 4 men and 2 boys can finish the same work in 2 days. Also 2 men and 34 men can finish it in $2\frac{1}{2}$ days. How long would it take 1 man, 1 woman and 1 boy working together at double their efficiencies to complete the work?
(A) $1\frac{4}{13}$ day (B) $2\frac{4}{13}$ day
(C) $3\frac{8}{14}$ day (D) $4\frac{8}{13}$ day

Q.171 Ajay invests Rs. 35000 in a bank at 7% p.a. How much more should he invest at 9% p.a. S.I. to get a 8% return on his investment?
(A) Rs. 32000 (B) Rs. 35000
(C) Rs. 30000 (D) none of these

Q.172 Mahesh had 300 debentures of company from which he received half yearly interest of Rs. 750. If the rate of interest is 10% annum. The face value of one debenture is
(A) 55 (B) 50
(C) 60 (D) none of these

Q.173 Preeti purchased six years National Saving Certificates for Rs. 1000. After six years she gots Rs. 2015. The rate of interest, if the interest is compoundly half yearly is:
(A) 13% (B) 11%
(C) 12% (D) none of these

Q.174 If the 2 incorrect watches are set at 12.00 noon at correct time for the first time that first watch gain 1 minute in one hours and 2nd watch losses 2 minutes in 1 hour:
(A) 12.00 noon, 15 day later
(B) 12.00 noon, 30 day later
(C) 1.20 am, 15 day later
(D) 12 am, 30 day later

Q.175 A dealer makes her goods at 125% of cost price and then allow a 4% discount for cash payment. Calculate his profit on cash sales:
(A) 10% (B) 15%
(C) 20% (D) 25%

Q.176 Yearly salary of a worker is Rs. 80,000 and one gift article what is the price of the gift article if after saving for 8 months he gets Rs. 50.000 and the gift article?
(A) Rs. 5000 (B) Rs. 7500
(C) Rs. 9000 (D) Rs. 10000

Q.177 A milk man has a certain number of milk animal one fifth are cows and their the square root of the total number are buffaloes. If the remaining five are goats the number of milk animals is:
(A) 25 (B) 16
(C) 244 (D) 15

Q.178 What is the average of the sixth seven number, fifth and eight odd number after 200?
(A) 210 (B) 211
(C) 212 (D) 213

Q.179 Ram and Raja work together for day $1\frac{1}{2}$ and earn Rs. 1.50. Ram works two times more than Raja in a single day. How much will Ram earn alone in one day?
(A) Rs. 2 (B) Re. 1
(C) Re. 0.66 (D) Re. 0.33

Q.180 The average weight of a class with 20 student is 48 kgs. If a new student weighing 60 kgs joins and an old student leaves the class, the, average weight drops down by 1 kg, what was the weight of the student replaced?
(A) 100 kg (B) 80 kg
(C) 60 kg (D) 120 kg

Q.181 If a is the simple interest on 'b' and 'd' is the simple interest on 'c' for the same period at same rate. The relation between a and c in terms of period 'p' and rate 'r' is:
(A) $a = \frac{brd^2}{100}$ (B) $b = \frac{c^2r^2d}{100}$
(C) $c = \frac{b^2c^2d}{r}$ (D) $a = c\left[\frac{dr}{100}\right]^2$

Q.182 Ram's efficiency is 30% more than his friend Shyam. There is a work to be done which Ram can do exactly in 20 days working alone. Ram works for 10 days and Shyam joins him and they together complete the work. The ratio of work done by Shyam to work done by Ram:
(A) 3 : 17 (B) 5 :17
(C) 7 :17 (D) none of these

Q.183 Cost price of two houses is the same. One is sold at 15% gain and the other for Rs. 2300 more than the first. If the net gain is 20%. The cost price of each house is?
(A) Rs. 20,000 (B) Rs. 22,000
(C) Rs. 23,000 (D) Rs. 32,000

Q.184 Vinod received some money by way of prizes for standing first is standard VII with this he pays his fees for 8th standard and deposits the remaining amount of Rs. 450 in a bank. Approximately how much will he get for paying his fees on admission to standard II if the bank pays him compound interest 10% per annum?
(A) Rs. 600 (B) Rs. 618.50
(C) Rs. 640.42 (D) Rs. 750.50

Q.185 A train covers a given distance moving at a speed of 60 km/hr. However if it was to halt for a first time every hour, its average speed come out to be 50 km/hr. For how much time does the train halt even hour?
(A) 5 m
(B) 10 m
(C) 15 m
(D) cannot be determines

Q.186 In a 800 metre race. A beats B by 40 m and C by 30 m. If they by what distance will C beat B in 9400 m/sec?
(A) $49\frac{24}{77}$ (B) $14\frac{10}{77}$
(C) $5\frac{15}{77}$ (D) none of these

Q.187 A boat sails 15 km of a river stretch up stream in 5 hours. How long will it take to cover the same distance downstream if the speed of current is one fourth the speed of the boat in still water:
(A) 1.5 hour (B) 1.8 hour
(C) 2.0 hour (D) none of these

Q.188 770 have been divided among A, B, C such that A gets (2/3) of what B gets (1/4) of what C gets then B's shall is:
(A) 120 (B) 160
(C) 240 (D) 320

Q.189 If $x = \frac{4ab}{a+b}$ the value of $\frac{x+2a}{x-2a}+\frac{x+2b}{x-2a}$ is:
(A) 1 (B) 2
(C) 4 (D) 16

Q.190 Mean proportional to 16 and 36 is:
(A) 24 (B) 30
(C) 26 (D) 81

Q.191 A man has some hens and cows. If the number of heads be 48 and the number of feet 140. The number of hens will be:
(A) 26 (B) 24
(C) 23 (D) 22

Q.192 In covering a distance of 30 km, Amit takes 2 hour man than Suresh, If Amit doubles his speed, he would take 2 hour less than Suresh, Amit's speed is:
(A) 5 km/h (B) 9.5 km.h
(C) 6 km/h (D) 6.25 km/h

Q.193 A and B each has some money if A give 30 to B, then B will have twice the money left with A. But if B gives 10 to A, then A will have thrice as much a is left with B then A and B have respectively:
(A) 34, 62 (B) 72, 44
(C) 62, 34 (D) 44, 72

Q.194 A number consists of two digits whose sum is 10, if 18 is subtracted from the number, digits are reversed the number is:
(A) 73 (B) 64
(C) 55 (D) 46

Q.195 Compounded ratio of 2 : 3 and 3 : 4 is:
(A) 1 : 2 (B) 2/3 : ¾
(C) 8 : 9 (D) none of these

Q.196 In a mixture of 60 litres the ratio of milk and water is 2:1, if this ratio is to be 1:2 the quantity of water is litre to be further added is:
(A) 20 (B) 20
(C) 40 (D) 60

Q.197 A bag contains Rs. 600 in the form of 1-rupee, 50 paise and 25 paise coins in the ratio 3:4:12 the number of 25 paise coins is:
(A) 600 (B) 900
(C) 1200 (D) 1376

Q.198 If $t_n = \begin{cases} n^2, \text{when } n \text{ is even} \\ n^2+1, \text{when } n \text{ is odd} \end{cases}$
find $t_{15} - t_{10}$ **[NTSE]**
(A) 116 (B) 126
(C) 106 (D) 226

Q.199 The 5th terms of the sequence defined by $t_1 = 2$, $t_2 = 3$ and $t_n = t_{n-1} + t_{n-2}$ for $n \geq 3$
(A) 13 (B) 15 **[NTSE]**
(C) 16 (D) 18

Q.200 The sum of the 4th and 8th term of an A.P. is 24 and the sum of its 6th and 10th terms is 44. Find the first term. **[NTSE]**
(A) 13 (B) 12
(C) –13 (D) –14

Q.201 The nth terms of an A.P. $\frac{1}{m}, \frac{m+1}{m}, \frac{2m+1}{m}, \ldots$ is:
(A) $\frac{m+1-mn}{m}$ (B) $\frac{mn-m+1}{m}$ **[NTSE]**
(C) $\frac{mn-m-n}{m}$ (D) $\frac{mn+m-n}{m}$

Q.202 If the numbers 3k + 4, 7k + 1 and 12k – 5 are in A.P. then the value of k is: **[NTSE]**
(A) 2 (B) 3
(C) 4 (D) 5

Q.203 An A.P consists of 50 terms of which 3rd term is 12 and the last term is 106. Its 29th term is: **[NTSE]**
(A) 58 (B) 60
(C) 61 (D) 64

Q.204 The 4th term of A.P. is equal to 3 times the first term and 7th term excess which the third term by 1. Find its nth term. **[NTSE]**
(A) n + 2 (B) 3n + 1
(C) (2n + 1) (D) 3n + 2

Q.205 If 5 times the 5th term of an A.P. is the same as 7 times the 7th term, then find its 12th terms. **[NTSE]**
(A) 0 (B) 11
(C) 14 (D) 18

Q.206 For what value of n, the nth terms of an A.P. is
(i) 63, 65, 67, ...and **[NTSE]**
(ii) 3, 10, 17, ... are equal?
(A) 10 (B) 11
(C) 12 (D) 13

Q.207 Which term of an A.P. 3, 15, 27, 39, ... will be 132 more than its 54th term? **[NTSE]**
(A) 1st (B) 63rd
(C) 65th (D) None of these

ANSWER KEY

1.	C	**2.**	A	**3.**	C	**4.**	B
5.	C	**6.**	B	**7.**	C	**8.**	D
9.	C	**10.**	A	**11.**	A	**12.**	B
13.	C	**14.**	D	**15.**	D	**16.**	C
17.	D	**18.**	A	**19.**	C	**20.**	C
21.	B	**22.**	D	**23.**	A	**24.**	C
25.	B	**26.**	C	**27.**	B	**28.**	A
29.	C	**30.**	D	**31.**	C	**32.**	B
33.	C	**34.**	A	**35.**	B	**36.**	C
37.	D	**38.**	C	**39.**	B	**40.**	B
41.	C	**42.**	A	**43.**	B	**44.**	C
45.	A	**46.**	C	**47.**	B	**48.**	B
49.	A	**50.**	C	**51.**	A	**52.**	A
53.	C	**54.**	A	**55.**	A	**56.**	A
57.	B	**58.**	C	**59.**	A	**60.**	B
61.	A	**62.**	B	**63.**	A	**64.**	A
65.	A	**66.**	B	**67.**	B	**68.**	B
69.	A	**70.**	C	**71.**	B	**72.**	B
73.	A	**74.**	C	**75.**	B	**76.**	C
77.	C	**78.**	A	**79.**	A	**80.**	A
81.	A	**82.**	D	**83.**	B	**84.**	B
85.	D	**86.**	D	**87.**	B	**88.**	A
89.	C	**90.**	C	**91.**	C	**92.**	D
93.	A	**94.**	C	**95.**	A	**96.**	A
97.	A	**98.**	C	**99.**	A	**100.**	A
101.	D	**102.**	A	**103.**	B	**104.**	B
105.	C	**106.**	B	**107.**	D	**108.**	D
109.	A	**110.**	A	**111.**	A	**112.**	B
113.	B	**114.**	A	**115.**	A	**116.**	A
117.	C	**118.**	A	**119.**	C	**120.**	C
121.	B	**122.**	C	**123.**	A	**124.**	B
125.	A	**126.**	B	**127.**	D	**128.**	B
129.	B	**130.**	A	**131.**	C	**132.**	A
133.	C	**134.**	C	**135.**	D	**136.**	B
137.	A	**138.**	D	**139.**	B	**140.**	C
141.	B	**142.**	C	**143.**	A	**144.**	B
145.	A	**146.**	D	**147.**	A	**148.**	C
149.	B	**150.**	A	**151.**	D	**152.**	B
153.	D	**154.**	A	**155.**	C	**156.**	D
157.	A	**158.**	D	**159.**	B	**160.**	B
161.	C	**162.**	D	**163.**	D	**164.**	D
165.	C	**166.**	C	**167.**	C	**168.**	B
169.	D	**170.**	D	**171.**	B	**172.**	B
173.	B	**174.**	C	**175.**	B	**176.**	C
177.	D	**178.**	A	**179.**	C	**180.**	C
181.	B	**182.**	D	**183.**	B	**184.**	C
185.	A	**186.**	B	**187.**	C	**188.**	B
189.	A	**190.**	B	**191.**	D	**192.**	A
193.	A	**194.**	C	**195.**	B	**196.**	A
197.	D	**198.**	B	**199.**	A	**200.**	C
201.	B	**202.**	B	**203.**	D	**204.**	C
205.	A	**206.**	D	**207.**	C		